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Remodelling Leadership: Quantum modelling of Wise Leadership Paper

DAVID ROONEY AND SUDP PATRA O P JINDAL GLOBAL UNIVERSITY
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David Rooney, Sudip Patra

Abstract

Since the late 90s a paradigm shift began in decision research that has implications for leadership research. Due to the limitations of standard decision theory (based on Kolmogorovian/Bayesian decision theory) scholars began to build a new theory based on the ontological and epistemological foundations of quantum mechanics. The last decade has witnessed a surge in quantum-like modeling in the social sciences beyond decisionmaking with notable success. Many anomalies in human behavior, *viz*, order effects, failure of the sure thing principle, and conjunction and disjunction effects are now more thoroughly explained through quantum modeling. The focus of this paper is, therefore, to link leadership with quantum modeling and theory. We believe a new paradigm can emerge through this wedding of ideas which would facilitate better understandings of leadership. This article introduces readers to the mathematical analytical processes that quantum research has developed that can create new insights in the social scientific study of leadership.

INTRODUCTION

We begin with three observations. First, quantum theory is argued to lend itself to developing new empirical insight into complex, indeed, indeterminate human behavior. Second, there is reason for the research community to take stock of how we approach leadership research so that we might develop wiser leaders who make more positive impacts on the world. Third, at its most fundamental level, wise leadership is an indeterminate social practice that is amenable to quantum social scientific investigation. By indeterminate, we mean that it is uncertain exactly what the best and wisest thing to do is as a leader as each leadership challenge unfolds. This is partly to do with the complexity of the human mind and behavior, and also the complexity and ambiguity of the context in which leaders act. Dealing excellently with this leadership indeterminacy, we argue, requires wisdom. Leadership and wisdom are both deeply situated practices that are necessarily shaped by context. This article shows that researching indeterminate social practices can benefit from the ontological and epistemological tools developed to understand quantum mechanics. Specifically, we show how quantum-like modeling can address researching wise leadership and ~~we~~ introduce the mathematical procedures that leadership research can employ.

The ontological assumptions in quantum theory are relevant to the indeterminacy of social behavior and, in particular, the impacts of social, cultural, economic, political and other contextual factors on behavior that create ambiguity, uncertainty, bounded rationality, imperfect knowledge, and so on. For reasons of economy, we will henceforth refer to the totality of these contextual factors as context. We suggest that better understandings of the way leaders practice leadership require researchers to examine any inadequate ontological assumptions in research designs and to consider the methodological implications that flow from a revised ontology, particularly in relation to indeterminacy and context. Thus, we will show that the ontology and epistemology inherent in quantum mechanics and quantum physics research (including quantum mathematical analysis) are useful for the indeterminate ontology of leadership[1]. Indeed, McDaniel and Walls (1997), Lord, Dinh and Hoffman (2015), Dyck and Greidanus (2017), and Hahn and Knight (in press) argue that quantum theory is a useful new lens with which to understand ambiguity, paradox, diversity, relationships, social interaction, time and change in organizations.

Importantly, quantum mechanics begins with "an undefined state, and offers an innovative approach for understanding the unfolding of complex organizational phenomena" (Lord et al., 2015: 264). Arguably, the most important message about the value of quantum theory to management and leadership research is, according to Dyck and Greidanus (2017), ~~is~~ that it offers us an alternative to entrenched ontological assumptions that no longer serve us as well as we would like. This article extends the early work of McDaniel and Walls (1997), Lord et al. (2015), Dych and Greidanus (2017), and Hahn and Knight (in press) by, firstly, linking it specifically to leadership and wisdom research through the mathematics and logic of quantum-like modeling (QM) as well as discussing examples of its use in behavioral economics, decisionmaking and cognitive science research where quantum modeling is already used. Secondly, by taking this body of research beyond making ontological and epistemological arguments we show how empirical research can operationalize quantum theory and quantum mathematical formalism. In this second respect, we introduce quantum logic and mathematics to assist quantitative modeling of leadership generally and wise leadership in particular[2]. If research is to contribute to developing wise leadership practitioners, then, a QM approach may be an important way to do this?

It may surprise some that the mathematical and logical tools for describing and predicting the microphysical or subatomic world (for example, Haven & Khrennikov, 2017; Khrennikova, 2017; Khrennikova & Patra, 2019) are useful in researching human behavior. However, just as calculus was invented for describing physical systems and was found to have significant utility in social science, so too, we argue, does quantum mathematics. However, we go further and argue, specifically, that QM is able to overcome significant limitations faced by those using standard quantitative social science tools that do not adequately link behavior and context, and lean too unrealistically on assumptions about linear and deterministic dynamics in what is a non-deterministic, uncertain, ambiguous, and even paradoxical world. "Quantum theory can represent multiple interacting paths through

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[1] However, we should caution readers that we would like to adapt the mathematical and logical framework of quantum theory only, rather than the physics of it. The emerging quantum like modelling in social sciences aims at that, for example, see Haven and Khrennikov (2013).

[2] A good coverage of extant emerging literature can be found in the research hand book edited by Haven and Khrennikov Haven, E., & Khrennikova, P. 2018. A quantum-probabilistic paradigm: Non-consequential reasoning and state dependence in investment choice. ***Journal of Mathematical Economics***, 78: 186-197..

time and, thus, can represent the complexity of change in ways that more conventional models cannot" (Lord et al., 2015: 265). In other words, we see that the underlying ontological assumptions in quantum theory are more realistic and have more empirical utility than those that underpin standard positivist (Hahn & Knight, in press) social science statistical analyses. Quantum-like modeling, therefore, can lift the impact of leadership research in an ever more intractable world. We do not, however, argue that the laws of quantum physics govern social behavior.

In mathematical (epistemological) terms, another reason for taking a QM approach is to extend its quantum probability theory framework to understanding the impacts of leadership and wisdom's contextuality~~[3]~~. "[Q]uantum mechanics is inherently probabilistic, rooted in the idea that all we can know about reality is the probability of experiencing a specific instantiation of it" (Hahn & Knight, in press: np.). This understanding of probability is based on the central role of context in quantum mechanics because the context, that is, a quantum system contains the full range of possibilities that the system can achieve. This system, therefore, is indeterminate and does not become something in particular until it is observed (that is, measured or experienced by someone).

It is salient for leadership research that the limitations of standard decision theory have been noted many times since the seminal works of Tversky and Kahneman (1992). Important findings have also been provided by QM in behavioral economics and cognitive science research (Haven & Khrennikov, 2013, 2017; Thaler, 1994). If leadership is done in an indeterminate context and wisdom is the human quality for delivering excellence under conditions of indeterminacy it is well worth considering QM[4].

we begin with a review of critiques of leadership research including of its ontological positions that problematic for good research design. We then introduce some foundational concepts in quantum mechanics followed by more detailed explanations of ~~,~~ quantum Beyesian (QBism) theory (Caves, Fuchs, & Schack, 2002), quantum field theory, and quantum decomposition theory. Having established a conceptual foundation, we then survey how QM is currently being used in cognition and decisionmaking research. We do this to give readers concrete examples of how to conceptualize research designs based on QM theory. Finally, we set out a range of research questions that QM can help solve. We have added a brief but relevant mathematical appendix setting out the basic mathematical tools of quantum theory and also a brief technical note on measures of entanglement.

Leadership

It is hard to avoid criticisms of poor leadership around the world (Clegg, e Cunha, Munro, Rego, & de Sousa, 2016; Dhiman, 2017; Tourish, 2013). Going a step further, researchers (Grint, 2007; King & Nesbit, 2015) argue that leadership training is ineffective at developing graduates who embody excellent leadership qualities that leadership theories call for. Leadership researchers are also interested in problematic kinds of leaders such as toxic (Pelletier, 2010), destructive (Schyns & Schilling, 2013), narcissistic (Rosenthal & Pittinsky, 2006), and psychopathic (Boddy, 2015) leaders, because the experience of being led in contemporary workplaces consistently does not meet minimum standards for ethical and professional practice. The weight of concern about poor leadership lead us to suggest that quantum modeling is an important new option for researchers to consider. Leadership judgment, thinking, decisionmaking, and behavior are done in and are products of uncertain environments (contexts). Furthermore, leaders, like all people, are boundedly rational and often appear to make what look like 'irrational' or rash decisions. But why is it the case that such problematic leadership continues and how can we rethink leadership research and theory from a QM perspective? Before exploring these questions, we discuss transformational, authentic and servant leadership theories to identify gaps that QM could fill.

Transformational, servant and authentic leadership are commonly used frameworks for leadership research. Since Burns (1978), transformational leadership theory has dominated research. Transformational leadership theory focuses on four dimensions: (1) idealized influence, or a leader's ability to inspire followers to identify with them; (2) inspirational motivation, excellence in communicating the leaders vision to followers; (3) intellectual stimulation, the ability of a leader to inspire followers to be innovative, take risks and to challenge assumptions; and (4) individual consideration, the ability of a leader to foster individuals to meaningfully meet their own needs. Transformational leadership focuses sharply on the individual qualities and capabilities of the leader (Zacher, Pearce, Rooney, & McKenna, 2014). There is a strong emphasis on individual behavior within group settings in the theory. Nonetheless, some research findings say that transformational leadership is a poor predictor of leader job performance (Judge & Piccolo, 2004).

Authentic leaders are self-aware and act in harmony with, their values, knowledge, and emotions (Harvey, Martinko, & Gardner, 2006), they are future-oriented (Luthans & Avolio, 2003), use balanced information processing (Avolio & Gardner, 2005), and (as a consequence of these dispositions) are concerned to make a positive contribution to the external world (Ilies, Morgeson, & Nahrgang, 2005). Authentic leaders, therefore, embody these characteristics of excellence (Reh, Van Quaquebeke, & Giessner, 2017). An authentic leader, like a transformational leader, has a clear set of laudable values, including courage, as well as the skill to negotiate the complexities of the workplace and their leadership role to enact such excellence. Little, however, is said about the background or context in which these leaders must be so wise.

Servant leadership theory focuses on the benevolence and selflessness of the excellent leader (Neubert, Hunter, & Tolentino, 2016). Servant leaders put others' (followers) needs and wellbeing ahead of their own (Van Dierendonck, 2011). Interestingly servant leadership blurs the boundaries between leader and follower. Servant leaders, then, are necessarily humble, compassionate and wise, and are not 'power-junkies'. Arguably, servant leadership theory presents the most idealized version of leadership but is the theory in which leader, follower and context are most integrated, making leader, follower and context difficult to separate analytically. While few will doubt the attractiveness of this kind of leader, little effort has been made to deal with developing research designs that adequately account for this interfolded/entangled leader-follower-context ontology[5].

Shamir, House, and Arthur (2005) argue that separating cause and effect is very difficult given the way leadership research is approached. Judge and Piccolo (2004) go so far as to say that meta-analysis shows authentic leadership and transformational leadership are largely overlapping and that they amount to much the same thing. Van Knippenberg and Sitkin (2013) argue that we should, in fact, abandon transformational leadership theory (Van Knippenberg & Sitkin, 2013), and by extension other similar theories. Going further Batistič, Černe, and Vogel (2017) say that conceptual progress is mostly being made by researchers on the fringes and that mainstream leadership research is overly focused on individual characteristics of leaders and insufficiently deals with the multilevel materiality in which leaders work. Examples of this fringe research that would be sympathetic to quantum ontology use phenomenology (Küpers, 2007, 2013) and eastern (Case, 2013; Yang, 2016) philosophical frameworks. As Neubert et al. (2016: 905) found:

> [T]he relationships of servant leadership with creativity and with patient satisfaction mediated through job satisfaction were moderated by organizational structure such that the associations were enhanced under conditions of high levels of organizational structure ... High levels of structure combined with high levels of servant leadership yield the highest level of satisfaction, while the lowest levels of satisfaction result from combining high levels of structure with low levels of servant leadership or low levels of structure with high levels of servant leadership. Alternatively, high levels of structure uniformly relate to lower levels of creative behavior, an overall effect that is buffered slightly with high levels of servant leadership. Together, the findings support the hypothesized effect of structure enhancing the associations of servant leadership with nurse job satisfaction and creativity, while also indicating that high levels of organizational structure suppress both outcomes in the absence of servant leadership.

In other words, leaders' impacts are clearly not only the sum of personal traits because context matters. Wisdom in leadership research is relevant to this discussion because it takes context seriously and de-centers the individual. Wisdom is also an indeterminate phenomenon. Integration, or harmonious interactions (Küpers & Statler, 2008) are important in wisdom as a social practice but these interactions occur in a messy political world of resource constraints in specific times and situations (McKenna, Rooney, & Boal, 2009). A wise leader is wise because s/he understands the quantum-like ontology of the life-world and is able to adroitly work with it (McKenna & Rooney, 2008). The complex dynamics that enable wisdom to be displayed in leadership practice through relational accomplishments and by overcoming the hindering pragmatics of life (Yang, 2011), and the impact of culture, history and political economy on present day leadership practices (Oktaviani, Rooney, McKenna, & Zacher, 2015) are discussed in the wise leadership literature.

Despite the multilevel/contextual complexity that leadership is practiced in, leadership research continues to take an individualistic focus. One result of this focus is that leadership theory continues to prosecute the idea of the leader who is something of a (moral) hero rather than a context constrained and boundedly ethical social agent, whose performance is determined largely by broadly sociological variables, and so, we argue, a significant theory-practice gap continues to thwart the impact of research on practice (Alvesson & Sveningsson, 2003; Learmonth & Ford, 2005). A different way of thinking about leadership research questions is exemplified in questions like what is the probability that "leadership emergence differs for males and females when they demonstrate the same pattern of behavior" (Lord et al., 2015: 280), which a QM approach can answer.

The dominant methodological paradigm in contemporary leadership research is a quantitative one that has relied mostly on classical probabilistic statistics and classical objectivist ontologies, and is based heavily on limiting but largely unwritten assumptions (latent variables and latent constructs) implying that context is static and unambiguous, and that self-report data adequately accounts for context. Indeed, explicit discussions of ontological assumptions are rare in quantitative leadership research articles. However, recent critiques of leadership research designs (Anderson, Baur, Griffith, & Buckley, 2017; Batistič et al., 2017) include the important observation that too much emphasis is given to single level designs and, relatedly, that context is not well handled by those designs. The purpose of this article, then, is to propose an alternative form of quantitative leadership research in the form of quantum (like) modeling (QM). We argue for this because of the different ontological assumptions that quantum theory makes and because of the ability of quantum modeling to operationalize those assumptions quantitatively in meaningful and powerful ways. A clear advantage of QM theory is that its predictions can be empirically tested and are presentable in numerical simulations (Haven and Khrennikov, 2013). Asano et al. (2017) show that applying quantum theory and empirical social science observations in context (e.g., in uncertainty) produces context driven frameworks that are more flexible in explaining non-trivial paradoxes like people making seemingly irrational choices that go against their or their organization's best interests[6]. The extant organizational research literature that advocates for the use of quantum theory has not yet explained how to use it to create new approaches to quantitative research.

Another important set of questions relate to how leadership teams function in, for example, making strategic choices. Recently, quantum decision theory has been applied to probability-based problems and the role of shared knowledge (Aumann, 1976). Such research considers how social agents in a group can disagree on posterior probabilities of events even though knowledge of prior events is shared by all group members and each person knows what prior beliefs that all group members hold. This literature uses common knowledge theory~~[7]~~, where every agent knows a specific event, or knows the probability of it happening, and also knows that everyone knows that everyone knows the probability of that happenstance. In brief, the event is, as it were, a public knowledge. Clearly, though, the assumptions stated here are unlikely to hold in most social situations. Aumann (1976) provides a set-theoretic structure of the theorem, where, if two rational agents start with a common prior belief about the event, and update their beliefs according to a Bayesian updating model, then reach a posterior degree of belief (represented by a probability measure, strictly speaking a Kolgomorovian set theory measure) about the same event where the posteriors are common knowledge to every agent, then there is no way that agents can disagree on the probability measures for the event. However, in workplaces this almost never happens casting doubt on Beyesian theory approaches to deal with social complexity

Coming back to reality, we often see examples in decisionmaking where even if common knowledge holds good, agents still disagree about their degree of beliefs, which, in turn, may lead to failure to achieve agreement. Thus, Quantum Decision Theory has been extended to understand 'disagreements' among agents, and demonstrates (Khrennikov, 2015) that when probability computation and updates are based on quantum theory, rather than set theory, different solutions emerge, where rational agents have freedom to disagree (or simply come to different decisions) while keeping common knowledge intact. This approach allows for more complex cognitive and decisionmaking processes than does standard decision theory. We provide some basic quantum ontology outlined above but we must now look deeper to unpack some epistemic insights that bring new possibilities of quantitative leadership research.

Quantum Basics

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[6] Such contextual utility models can show various effects like preference reversals, ambiguity aversion or attraction, all embedded in a single coherent framework. Our point is that a single coherent framework is critically needed in leadership decision theory also.

Quantum theory is useful to explain human action "by adopting a process-oriented approach that attempts to understand how different presents are actively created" from the range of potential outcomes that a context allows (Lord et al., 2015: 269). Quantum reality is set in a context of relationships and interactions between many variables.

A very important ontological feature of quantum theory is that it deals with what is called ‘deep uncertainty’. In quantum theory, the fundamental or pure state of any system is represented by a ‘superposition’ (the sum of all interacting variables in a system prior to taking any measurements/observations, i.e., probability) of basis states. This ‘pure’ superposition is the context out of which emerge the events we experience as our social reality. This emergent subjective experience is called a mixed state. The potential for all basis states is contained in the Hilbert space superposition. The contents of a Hilbert space superposition interact with and influence each other creating a large number of potential outcomes from those interactions. This interaction process is called entanglement. Entanglement is a state in which “two or more quanta interact to form a composite superposition that results in a new, single quantum entity” (Hahn & Knight, in press: np.). In an organization, Hilbert space is very much a superposition of intersubjectivities, interacting in shifting patterns of relationships where the impact of interactions is fundamentally uncertain or indeterminate (McDaniel & Walls, 1997: 369) because:

> In the quantum world, the problem is not that we do not have enough information about the present state of affairs or even the past state of affairs to predict the future. No matter how accurate or complete our information is, the world is fundamentally unknowable … When we try to know the world, particularly through measurement of it states, we come face to face with the Heisenberg uncertainty principle that says that if you measure position accurately , you must sacrifice and accurate knowledge of momentum (Herbert, 1985, p. 68). Every attempt to know one attribute of a system reduces our ability to understand other attributes; this leaves us with a world that we can never completely know.

Most importantly, quantum uncertainty does not vanish with the addition of more information.

It is important for this article that Aerts et al. (2013) have summarized a two decades of research on the correlation between quantum theory and human decisionmaking and cognition. These researchers say that just as quantum theory’s measurement entities (for example, observables like position, momentum, or the energy of particles) are influenced by the context of the measurements (measurement apparatus or the measurement environment as a whole) and deep uncertainty, so too is human cognition and decisionmaking. Aerts et al. (2013) have also demonstrated that quantum like correlation (known as entanglement) exists in human decision states. For example, one person’s belief state interacts (is entangled with) other peoples’ belief states. We can also explore much more challenging aspects of human life using QM. In one, study (Dalla Chiara, Giuntini, &

Negri, 2018: 78) the semantics of poetic and musical metaphor expressed in songs was conducted to understand how extra-musical meanings are created by interlacing musical ideas in the musical score and poetics devices in the lyrics as an example of quantum emergence from an indeterminate context. They argue that;

> [A]n important feature of music is the capacity of evoking extra-musical meanings: subjective feelings, situations that are vaguely imagined by the composer or by the interpreter or by the listener, real or virtual theatrical scenes (Dalla Chiara et al., 2018: 79).

A formal analysis using a quantum approach is possible because:

> As happens in the case of composite quantum systems, musical ideas (which represent possible meanings of musical phrases written in a score) have an essential holistic behaviour: the meaning of a global musical phrase determines the contextual meanings of all its parts (and not the other way around) (Dalla Chiara et al., 2018: 78).

Given the complex and nuanced intersubjective dynamics that are necessary for excellence in leadership (Küpers & Pauleen, 2013), there is clearly a place for a quantum approach in leadership research. Indeed, the logic of QM indicates that quantum theory's mathematical and logical framework is very adaptable for social science. We believe leadership is a fertile ground to which to extend QM because of the indeterminacy of it as a practice and because we need to understand how to foster excellent or wise leadership by working with rather than against its deep uncertainty and unknowability.

Classical and Quantum ontology

To reiterate, we are not proposing a physical theory of quantum leadership. However, there is a growing awareness (Haven, Khrennikov, & Robinson, 2017) that the mathematical, logical and ontological structures of quantum theory are compatible with the realities faced in social action, and this article extends this view to the deeply complex phenomenon of leadership.

The basic conflict of world views between quantum physics and classical physics lies in conceptions of probability and locality. In classical, deterministic physics (from Galileo to Einstein) probability is understood to be, at best, a secondary concern and arises in classical thinking because the experimenter has an incomplete set of information about the underlying variables that create the external world. Relatedly, randomness and uncertainty are not central aspects of classical scientific ontology. Underpinning this assumption is the additional assumption that if one has full knowledge of reality everything is predictable.

Quantum theory, on the other hand, interprets nature as fundamentally random; that is, there is an irreducible randomness to the universe, which is described by what is called uncertainty relations, where uncertainty remains no matter how much information we have~~[8]~~ (Birkhoff & Von Neumann, 1936). Uncertainty in quantum theory is deep, and refers fundamentally to the superposition principle, where before a system is measured/observed it is in a superposed state of possibilities: only measurements/observations can alter the superposition and 'collapse' or crystallize it into a state that we observe as reality[9]. Importantly, randomness is, therefore, what defines uncertainty in quantum theory.

A related debate is the measurement problem debate. Classical physics assumes that a deterministic model of uncertainty underlies everything in the universe but that complete knowledge of the model is lacking and is yet to be discovered. In this paradigm, experimenters, therefore, have to hypothesize a hidden or latent and unmeasurable variable (as an assumption), which they place in their underlying model of reality. Each successive study can (hopefully) yield information on ever-increasing numbers of hidden variables underlying the deterministic reality. That is, a more complete description evolves as each hidden variable is discovered to gradually complete our knowledge of reality. Even chaos theory uses such deterministic philosophy, to explain what happens prior to the point of emergence. Quantum ontology, therefore, is, in many ways, the opposite to the classical, deterministic view. Social scientists also use this deterministic assumption to build their research designs that state assumptions and set out testable hypotheses. But what if these assumptions are flawed?

J.S. Bell (1966) proposed that if hidden variables do exist, then certain inequality correlation relations among random variables in a quantum experiment should work according to standard physics theory. Bell's experiments showed that quantum level behavior does violate the inequality relations in classical physics. Empirical validations of Bells inequality results have, therefore, ruled out hidden variable theories~~[10]~~ in quantum mechanics and raise interesting challenges for how we understand the role of information and knowledge in research design.

Bell's inequalities says that no hidden or latent underlying structure of reality exists and, therefore, that the predictive ability of quantum theory is not hindered because it does not use latent variables. In a social science context, where latent variables are commonly used, this is a potentially ground-breaking change for research design.

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[9] In quantum physics there are stylized uncertainty relations, for example, the product of momentum and position uncertainty measures are greater than or equal to h/2π, where h is Plank's constant. In social science we can refer to the superposition description readily, for example, as in quantum decision theory, where deep uncertainty is described by the superposition of beliefs, which is defined in terms of density matrix operators (we present more detail on this formalism later).

Bell type inequality relations are readily observed in human decisionmaking experiments (Dzhafarov & Kujala, 2016), and are clearly linked to the influence of contextuality in cognition (Aerts et al., 2013). Another implication of Bell type inequalities is that ‘entangled’ states exist between random variables. Very simply put, entangled states are composite states of at least two entities that cannot be conceived of separately (Aerts et al., 2013). Entangled entities are, for examples, two or more particles that need to be related to create some aspect of or entity in the observed world, and which interact with (or react to) each other to create this entity (Hahn & Knight, in press). If one were to alter one of these entangled entities, then all the other entities that are entangled with it will react instantly to the change and “take complementary states depending on the measurement of the first entangled element, as if they “knew” what type of measurement was performed … [and this] implies that single elements of an entangled system cannot be fully described individually, but bear properties that depend on their interaction with other elements and the properties of the overall system”. (Hahn & Knight, in press: np.). Two or more objects that are correlated such that they interact with each other’s behaviors are entangled. Moreover, entangled entities do not need to be close to each other in time and space to influence each other. Thus, a person’s memories of being in a serious mid-flight emergency ten years ago on the way to Greenland is entangled with their decision today to not fly to a South Pacific Island for a holiday next year. This decision (or measurement/observation) is a result of ‘objects’ in the mind that are separated by many years and long distances but nevertheless are entangled (correlated). It is easy to imagine that complex patterns of relationships will emerge through entanglement. An outside observer can have full information about the system as a whole (i.e., the probabilities of mid-flight emergencies on long haul flights), but the sub-systems (the information that the decider used and their interpretation of it in relation to next year’s holiday) is at a random state. In purely rational, statistical terms, the decision not to go to the South Pacific is irrational and some other people with the same experience might also make the same choice but others will make a range of other very different choices. We might more accurately say that it is an emotional or anxious decision rather than irrational and that more information will not necessarily change that decision. Physicists describe entanglement as beginning with the larger system that exists in a ‘pure’ state of infinite probabilities and subsystems that exist in ‘mixed’, finite states. Physicists use the term pure state because this state is a superposition of the basis states in the given Hilbert space that contains all possibilities. We describe entanglement in greater detail later, but it is important to note that entanglement is often understood as the most striking difference between classical and quantum ontology but should be less controversial in social science. Entanglement allows deeper correlations between sub-systems than is allowed in classical probability theory~~[11]~~. Leaders are deeply entangled in quantum-like systems and are parts of many patterns of relationships.

Quantum modeling ontology assumes that reality is constantly changing and ambiguous, and that this indeterminate background or context is the platform on which the unique events of reality are enabled (or not). For example, if human behavior is based on each person's 'belief state' and the ways in which an individual person's belief state interacts (or is entangled) with the collective belief state of society will collectively produce specific behaviors. Many social scientists are comfortable with this ontology of becoming and intersubjective sociology (of knowledge). People do not often make decisions in everyday life based solely on fixed preferences. However, revealed preference theory holds that deviations from time invariant or context invariant preference patterns are irrational. Quantum decision theory focuses on context specific utility maximization (Aerts, Haven, & Sozzo, 2018), and hence more variance, uncertainty and randomness in preference patterns.

To pull these core quantum ideas together, we can say that quantum probability, entanglement, inequality, interdependence, randomness, and uncertainty are aspects of reality that decisionmakers and leaders face. In this regard, quantum theory and otology can contribute new ways to do leadership research by meaningfully accounting for this messy reality. For example, how was President Trump's belief state able to interact (entangle) with the aggregate of American society's collective belief state to make him President is a question quantum modeling can seek answers for by using quantum probability, entanglement, inequality relations, interdependence, randomness, and uncertainty.

As we stated above, in quantum ontology, randomness is held to be an intrinsic part of reality, which means that randomness or uncertainty is not produced by incomplete knowledge, it is a state that is independent of knowledge. Thus, even with complete knowledge (if that were possible) randomness remains. The questions arising from this situation for research design is what do we do in place of standard quantitative methods and that relies on randomness and entangled variable in a context.

A very important point to make at this juncture, is that a person's reality is emergent and personal. By this we mean than a person is an observer taking his or her 'measures' or observations to create the information, meaning and knowledge that they use to navigate life. Social reality is, therefore, emergent. The reality we experience emerges from Hilbert space upon our observation. In Figure 1 we show the observer with a pure Hilbert state space (above) of potentially infinite probabilities of quantum reality, which, in quantum parlance, 'collapses' (below) into an event (or interaction) within the mixed and finite state space of social reality. The observer is in this sense an interface between the two state spaces bringing an emergent social reality to life (in the bottom triangle) as he or she observes an aspect of the entangled pure Hilbert state space.

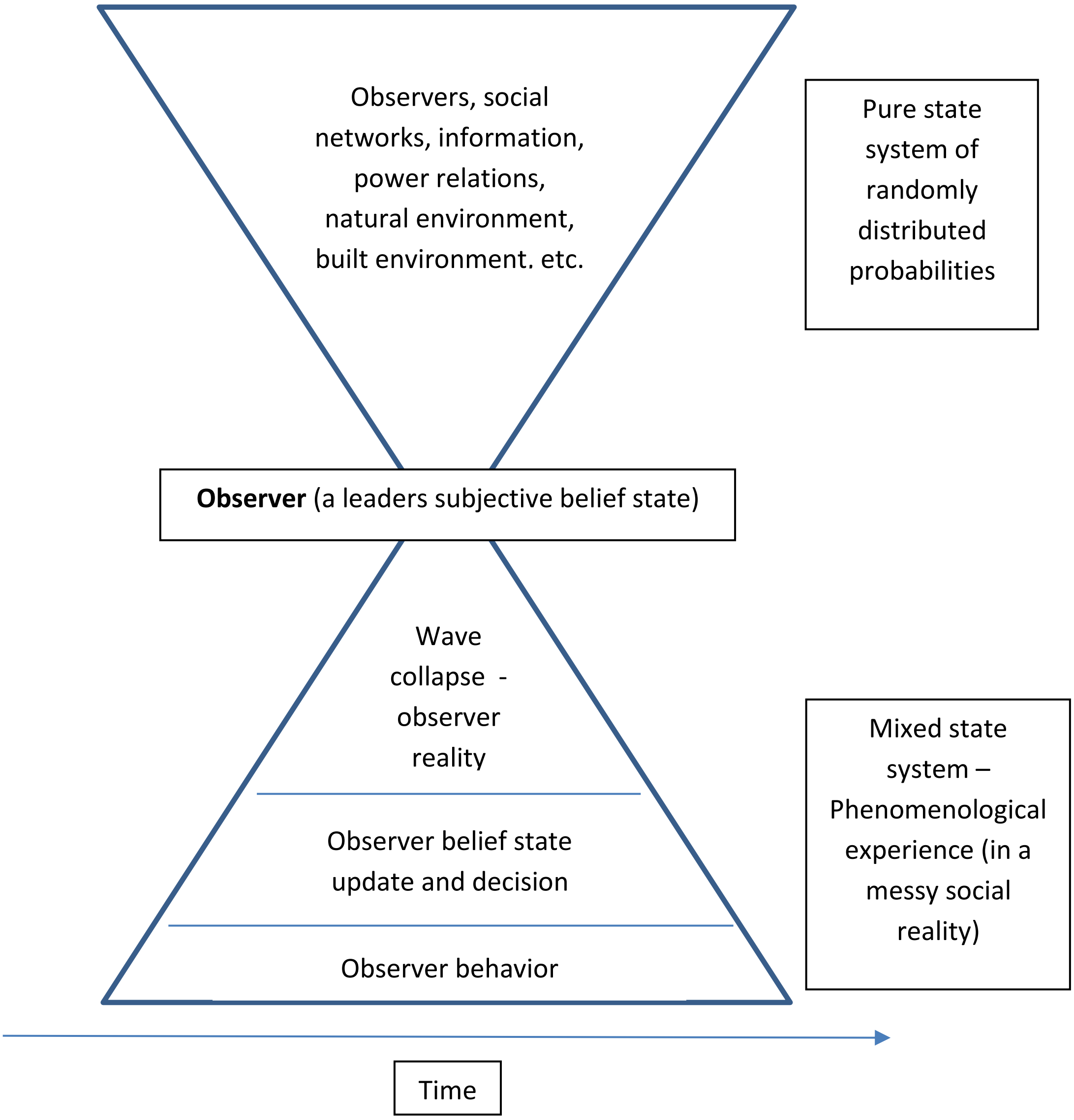

Figure 1. The position of an observer in state spaces

Readers must keep in mind that is diagram is an over simplification because we make observations regularly across time and in the presence of other observers. However, for the sake of clarity, we present Figure 1 as a single observer at a single point in time. We now shift the focus to specifically social scientific applications of QM.

Quantum Modeling: Probability & Subjectivity

We now discuss how probability and subjectivity work in quantum epistemology. Both probability and subjectivity work in different ways in quantum research compared to classical scientific epistemology. These important epistemic differences are, however, very useful for understanding social phenomena, and leadership in particular.

Recently, as summarized in Khrennikov (2015), researchers have used quantum probability theory to model human decisionmaking. This research shows that human information processing and, indeed, the mind are non-deterministic because they are contextual and adaptive. Consequently, probabilistic information processing cannot be well described by standard models of probability. However, the Vaxjo approach was developed to mode this indeterminate process (Haven & Khrennikov, 2013). Vaxjo theory[12] uses additive perturbative terms. Additive peterbative terms are an extension of classical total probability expression. An example is as follows (a more detailed mathematical approach is presented in appendix 2)[13]. Let's assume we would like to predict the total probability of event A happening given event B, where B is dichotomous to A and therefore has two values B1 and B2. Based on standard Bayesian probability theory we would express it as: $P(A|B)= P(A|B=B1)+P(A|B=B2)$, however the quantum probability the expression would be: $P(A|B)=P(A|B=B1)+P(A|B=B2)+2\rho(P(A|B=B1)P(A|B=B2))1/2$ where the additive terms are perturbative. Specifically, ρ is a phase angle (as elaborated in the appendix). Again, since quantum theory is used to derive such a formula, the result resembles the quantum formula for total probability, which is well established in quantum mechanics (please see appendix for a detailed working).

Quantum Bayesian modeling (QBism) (Caves et al., 2002) is another approach. It attempts to interpret the basic quantum state of any system as subjective and contextual, hence its probability measures are also subjective. There is a subtle difference between Vaxjo type interpretations of QM and QBism's interpretations. While the former uses statistical interpretations based on average results of an ensemble of identical states, the latter is concerned with how information processing and decisionmaking happens rather than the mean results. In other words, a personalist version of information processing and measuring probabilities is used in QBism. Thus, Khrennikov (2015) proposes that QBism is a general decisionmaking model. However, there are complexities in computing the formula for total probability in that model. QBism agrees with the personalist Bayesian probability theory as pioneered by Ramsey (1997), De Finetti (1974) and others. De Finetti (1974) suggested that there is no such thing as probability because there is only a personalist degree of belief. In this approach 0 and 1 probabilities are degrees of beliefs. However, QBism holds that the measurement outcome is not pre-existent, rather it is created in the act of measurement~~[14]~~. In social behavior there are numerous instances (Yearsley, 2017) where measurement

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[12] Vaxjo interpretation of the modified formula for total probability has emerged out of efforts by scientists at Vaxjo conferences on quantum foundations since last twenty years (Khrennikov, 2003).

[13] The formula presented here is originally motivated by the superposition principle in quantum mechanics, as discussed in the paper, and this formula famously appears in the probability computation of 'double slit experiment'in quantum physics, which is so well emphasised in Feynman lectures on physics Volume 3.

st interpretation of quantum theory, hence its proponents expect quantum theory to be a decision making theory with special axioms and rules.

effects are observed; for example, order and conjunction effects, and disjunction fallacies, which we discuss later in the paper. Thus, Chinese whispers-like errors can arise in decisionmaking processes as interactions between agent’s (belief states) within the information context unfold. Inaccuracies in information transmission will likely occur and this process can be described effectively by Quantum formalism[15]. But what does this formalism look like in social science research practice?

Using Quantum-Like Modeling in Social Science

All the QM examples that follow are based on research on cognition, decisionmaking behavior under uncertainty, or decisionmaking in specific contexts. Each of the examples has implications and applications for leadership research. The basic idea we explore in this section is that in quantum theory the context of measurement of any property influences the outcome. This is important for this article because social, political, economic, cultural and other contextual factors deeply influence social behavior and the ability to realistically model such complexity~~’~~s’ impacts on behavior are important.

In much of the social sciences, when contextual behaviors deviate greatly from an ideal, for example, when leaders practice narcissistic leadership rather than, for example, servant leadership, those behaviors are considered deviant, irrational or foolish outliers. However, QM provides explanations for such behaviors based on a set of ontological assumptions that render these ‘outliers’ as natural parts of reality, even if they are less than desirable. Clearly, some leaders are eccentric to the point of deviant but there are so many examples in history of undesirable leader behavior that research should not ignore ~~them~~ because they are, in fact, part of what is within the range probable (if not good) leader behavior. Moreover, unwelcome leader behavior is something that we need to know more about. Although we certainly see enough unwelcome or unsavory leader behavior in the world, the questions remain, what responses should we make to it and why does it keep happening? We still do not have good and actionable answers to such questions. We, therefore, now turn to discuss empirical QM research approaches to some specific context-shaped behaviors that are relevant to leadership beginning with the sure thing principle and uncertainty avoidance.

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[15] In quantum theory measurements are described by projection operators, or projection postulate, act of measurement is equivalent to projections of the initial superposed state into a definite Eigenvalue, probability of such a projection is provided by the Born’s rule. Such projection operators live in the Hilbert space of the system and are orthonormal to each other. There are other projection operators which are named as positive operators, which describes ‘unsharp’ measurements. In decision theory terms, orthogonal projection operators will project the intial belief state to a specific final state immediately after the measurement (for example immediately after a question is asked, where the act of asking question is measurement), where as a positive operator will project the initial superposed belief state into an unsharp state, for example ‘may be’ type of response.

Non-Optimal but Normal Behavior

The sure thing principle is a central assumption in standard decision theory. Simply put, it means that rational, utility maximizing agents do not include irrelevant information while making decisions. This, of course, is an unrealistic assumption because people use irrelevant information quite often. For example, take Bob, who is deciding to buy a house. In one scenario, his information set contains the information that Alice will win the next presidential election, and because of this, Bob decides to buy. In the second scenario, the information set contains the information that Alice will lose the election, and Bob still decides to buy the house. In this example we might conclude that Alice's state does not affect Bob's decision at all. However, using QM, Haven and Khrennikov (2013) show that under uncertainty, for example, when there is no information at all about the election, Bob may behave differently. Thus, in the face of uncertainty about the outcome of the election, Bob decides not to buy. In standard decision theory, such a behavior is considered irrational, but without any deeper explanation. Such behavior might be irrational, but it is quite normal in uncertain conditions.

Going further, and similar to Bob's response to uncertainty, in standard decision and game theory, the 'irrelevance of irrelevant alternatives' principle is well known. In the prisoner's dilemma (Rasmusen, 2007), for example, strategy equilibrium theory suggests that irrespective of what the other player chooses, the first player should always choose to defect; that is, not to cooperate. However, real people behave differently when the same game is played in an uncertain context; for example, when players have no idea about the move of other players or the players feel a sense of loyalty to the other players. Haven and Khrennikov's (2013) QM research lists many 'deviant' or non-optimal behaviors of players that cannot be described by standard decision theory mathematics. QM's Quantum Decision Theory (QDT) looks at these examples from a perspective that enlarges our typical utility framework to accommodate normal but non-optimal behaviors?

More important, however, is the difference between standard decision theory models and QDT is that the total probability formula (FTP) is different in QDT. In QDT the additional perturbation term modifies the FTP to account for the impact of contextuality (Khrennikov & Haven, 2009). Hence, contextual factors are quantifiable in a probabilistic sense and the 'non-optimal' behavior of agents under novel contexts such as uncertainty are meaningfully measurable. Leaders constantly face uncertainty and dilemma and so including the impacts of uncertainty in leadership research is important. Even though prisoners' dilemma-like scenarios abound in a leadership contexts, the failure of the sure thing principle suggests that people behave non-optimally, and frequently do not choose the cooperation strategy. Most importantly, Haven and Khrennikov (2013) show how decision states of agents can be reconstructed based on a QM theory framework that more realistically models complex social behaviors for large ensembles of people. Using such methods in leadership contexts, we could explain or even predict, for example, co-operation among leaders, or leader and followers in scenarios where standard game theory would fail to capture the social and emotional complexities of reality.

Hence, leadership is a fertile ground in which to test QDT predictions, more specifically we can deliberately include so-called irrational behaviors by leaders in analysis. Quantum modeling also allows research to do more complex studies of leader social cognition.

Order Effects in Human Cognition

Order effects research aims to understand how the order in which things happen to someone influences their choices. Essentially, researchers observe how peoples' responses to questions differ when the questions are asked in different orders (Haven and Khrennikov, 2013). However, the effects of different orderings are not simple. Bruza et.al (2015) who have pioneered the use of QM modeling to explain order effects in cognition show that if questions are asked in random orders~~,~~ and the questions have positive operator representations (each question is unrelated to each of the other questions), then it is not automatic that such operators will commute. In other words, if a change in the order of operators (questions) produces different output states (answers), we would conclude there is an order effect[16]. Further, if the questions are represented by non-commuting operators (that is, they have mutually complementary semantics), responses by agents will differ if the order of questioning changes because each question is semantically conditioned by (entangled with) each of the other questions. The implications of these kinds of order effects are potentially profound, yet we know little about them in leader behavior. It is useful to explore the mathematical logic that QM research uses to understand order effects.

For social science research, it is important that the measurement process in quantum theory occurs in two stages. First is a preparation state, which corresponds to the initial belief-state of a person, such as a leader. The preparation state (of beliefs) is represented by density matrices. In mathematical language, a density matrix is a description of information. In our case, information captured about a person's belief-state, or an ensemble of peoples' belief-states, which are represented as direct products (Tensor products) of initial matrices representing the pure states: $\rho=|X><X|$, where X is the initial pure state of belief, which can be thought of as a linear superposition of basis states, say $|X> = a|0> +b|1>$, where 0 and 1 are the basis states of the underlying two dimensional Hilbert space in this example, which may mean, for example, a down and up state for any future event happening, and $|a|^2$ and $|b|^2$ are the probabilities/ degree of beliefs (according to a quantum probability frame work) of such occurrences. Here, if the modulus square of a and b is ½ each, then both 0 and 1 states are equally possible, hence, this superposition reflects ignorance about the system.

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[16] Readers can be referred to a formal mathematical literature on the commuting and non-commuting observables or questions in decision theory, see for example, Bagarello (2019).

Importantly though, 0 and 1 states are symbolic because there can be as many states as the number of basis states in state space (that is, Hilbert space), and this superposition description is actually based on knowledge about the possible states before we measure in the next stage. For decision theory purposes the superposition state of beliefs is the state the belief system is in before any measurement is performed (i.e. before dealing with any questions related to a person's beliefs). There is a fundamental 'uncertainty' when in this unmeasured/uninterrogated superposition state.

Belief states can change, that is, they are updated over time, and so a model of belief states needs to measure updating. Researchers can ask participants to answer a question regarding 0 or 1 states, where any dichotomous choice variable in an experiment is represented by 0 or 1. For example, 0 and 1 can be belief states of agents in a market, where 0 is the belief that an asset's price is decreasing and 1 is the belief state that the asset's price is increasing (Khrennikova and Patra, 2019). Based on participants answers, we update their belief states. This is done by making the superposition state collapse to 0 or 1 by recording participants answers to questions. In more technical quantum theory terms, then, there are two phases; first, is the state prepared for experiment, say the belief state of the agents before they face questions, and, second, a random 'collapse' state that results from the act of measuring (observing/answering) the initial 'superposed' state causing it to change immediately to its final state (0 or 1). Probabilities are ascertained from observed frequencies. In leadership decisionmaking contexts, this could correspond to a cognitive experiment (Dzhafarov & Kujala, 2016) where the leader provides questionnaires to respondents, and probabilities of respondents answers are found by calculating frequencies of choices by the respondents. The mathematical psychology literature (see Yearsley, 2017) has numerous relevant experimental design examples.

The collapse postulate is geometrically described as an action of a projection operator (in our case, a question) on the ρ. However, these projection operators are orthogonal and form a complete orthonormal (scalar products of ith and jth projection operators =0) set in the given Hilbert state space.

Going further, in real decision scenarios, there is always noise in the system, which means decisionmakers may erroneously choose, say, the option 0 when they actually believe 1. Such errors in decisionmaking cannot be captured by simple projectors, hence a positive-operator valued measure (POVM), or positive semidefinite projector is used to capture any 'error-prone' or imperfect decisionmaking (Yearsley, 2017). Hence, if questions are represented by non-commuting operators, it is not difficult to see how the final output states or responses by agents will differ when the order of questions changes. Non-commuting operators are operators that represent the observables that cannot be observed or measured simultaneously with indefinite precision; for example, where [A,B] is non 0, or where [A,B]=AB-BA. Commutation relations are building blocks of any operator theory. The above bracket is known as the commutation relation between two operators, A and B. A and B in quantum theory represent observables; for example, A can be a position operator and B can be a momentum operator. In classical logic, AB-BA can only be 0 or, in other words, both observables can be observed simultaneously. However, in quantum theory this assumption can be relaxed. In quantum physics the non-commutation relation is famously expressed through Heisenberg's uncertainty relations between conjugate variables like position and momentum of sub-atomic systems. We can think of any pair of random variables representing different tenets of leadership, which may not be compatible with each other, for example, unethical and ethical leadership styles.

In the case of decisionmaking models, operator representations of observables to be measured must be built from scratch. In QDT these operators represent the questions which when asked to change the belief states of agents from their initial belief state and make them collapse to a new belief state. In mathematics, this kind of operation is called an Eigen value or Eigen state link. However, there are still challenges such as, for example, when questions are repeated? Will there still be an order effect (Aerts et al., 2018)? The implications of order effects are potentially profound, yet we know little about them in leader behavior. Beyond leader decisionmaking, order effects have implications in, for example, leader communication and how it influences other decisionmakers in an organization.

Standard order effects theory predicts that agents will choose different options if the order of questions is changed. However, the findings from QDT shows that because changing the order of questions also changes the context in which behaviors happen, it is not just question order that drives change but also the changed context. Mathematically, operator representations of observables (which do not commute with each other) are an elegant way to analyze such change in behaviors. However, we can tease more mathematical insight out of the quantum perspective by discussing conjunction and disjunction effects.

Conjunction and Disjunction Effects

Based on probabilistic behavioral models, we find regular violations of standard probability axioms, for example, $P(A\&B)> P(A)+P(B)$, the conjunction fallacy, or the opposite of it~~,~~ (the disjunction fallacy), i.e. $P(AUB)<P(A \text{ and } B)$ A and B being two events. The Kolgomorovian (Khrennikova, 2017) measure theory also does not accommodate such violations. However, if belief states are described by the superposition of basis states in Hilbert space, and measurements are represented by projections onto specific Eigen sub-spaces, and the probabilities of actualizing one final state (a behavioral outcome) is given by Born's rule, then such probability inequalities can be justified. Sequential choices can then be described by sequential measures/ projections. The implications of conjunction and disjunction effects in leadership are significant. For example, what is the relationship between doing management and doing leadership? And what is the difference between understanding

leadership as a role, a position on an organizational chart, and leadership as a way of being? We are still unclear on such effects. Hence, we can think of variables as A's and B's, as in the above expressions, and then give scores to the variables based on a Likert scale. Having done this, we can study the correlations between variables. Any conjunction or disjunction effects as expressed via the joint probabilities might throw light on how various aspects of leadership are perceived by agents in an organizational context. In the leadership context, we need to have variables that reflect and measure A/B (doing management = managerial observables, or doing leadership = leadership variables or attributes), which can then be provided as a questionnaire with a Likert scale for responses. Response frequencies can be used as probabilities, which can then be used to detect conjunction and disjunction fallacies. Such experimental designs might provide new insights into how leaders' influence their organizations and vice versa.

Heisenberg-Robertson Inequalities

Importantly, Heisenberg's uncertainty relation in the form of Robertson inequality is used to quantify uncertainty in ~~human~~ decisionmaking (Pothos & Busemeyer, 2013). As mentioned above, uncertainty is a challenge; for example, risky situations are used as proxies for uncertainty, but this is inadequate. Risky situations are situations with known or subjective probability distributions, whereas ambiguous or uncertain situations are where such probability computations are non-trivial. One example is the Ellsberg two urns paradox (al-Nowaihi & Dhami, 2017; Ellsberg, 1961). There are two urns of red and blue balls, in one of them the proportions of red to blue are known and in another the proportions are unknown. Participants have to place bets on what color ball they will take out of an urn. When agents are asked to choose one of these urns they tend to choose the urn with known proportions over the urn with unknown proportions. Such behavior is known as ambiguity aversion and is covered by the Bob and Alice example (above). To explore ambiguity aversion, mathematical psychologists (Pothos & Busemeyer, 2013) have used Hermitian operator representations of incompatible questions asked to respondents. In such models, mental states are dependent on the mutual uncertainties of incompatible questions. In these models, questions are represented as operators: as self-adjoint projector operators, and the actions of such operators are used to predict the mental states of participants as Eigen sub-spaces of the initially superposed belief states.

To recapitulate, cognitive quantum-like modeling provides a Hilbert space representation of belief states, where the belief state is considered to be a normalized vector in state space, or a general density matrix representation that shows a mixture of different pure states. Hence, it is possible to use Heisenberg's uncertainty principle to describe state space distributions. Recently some inequality relationships have been studied (Bagarello, Basieva, Pothos, & Khrennikov, 2018) that reflect the behavior of agents under uncertainty. Such inequalities can be used in leadership decisionmaking, since leadership is always exposed to uncertainty as they act within their context: a context that can both enable and constrain behavior.

Contextuality and randomness

We need to look more closely at context to properly understand QM. Dzafarov and Kujala (Dzhafarov & Kujala, 2016) have modelled contextuality in human behavior based on analogical mathematics within quantum theory. Contextuality models are particularly effective when outcomes are binary~~,~~ and some factors in the measurement context interact with the measurement process to influence the outcome. For example, in the cognitive experiments we have already discussed, if the order of the questions is changed, or new questions are asked along with the target question, responses vary widely. A famous example is Linda, the bank teller. When questions like, is Linda a feminist? are added in different orders with other questions, research participants answers are different because the context has changed. That is, the research participant can be directed to draw on different contextual elements in Hilbert space that influence their answer. Other questions that draw the respondent's attention to the history of women in traditionally male work domains (like banking) may elicit answers of yes, Linda is a feminist.

Recently, researchers (Basieva & Khrennikov, 2017) have measured contextuality in human decision data. In behavioral experiments, responses to questions in different contexts can be treated as random variables. Researchers have found two types of influence on the distribution of such random variables (i.e. the probabilities for the random variables attracting yes/no responses). Direct influence is observed when change in the distribution of one response changes as the context is varied. However, when direct influences are eliminated from the experiment, residual 'true' or deep contextual effects may be brought to light. Dzafarov and Kujala (2016) have demonstrated such contextuality in decision making.

This is a fast evolving area of research and leadership is a fertile ground for future empirical tests of the contextuality hypothesis. For example, do leaders make better decisions when in their office or when they are traveling in foreign countries? Also, how does leadership decisionmaking change when the information environment changes? For example, when a leader formally studies business or leadership and, therefore, is exposed to business school information environments, do they change significantly? Later we provide a simple quantum field theoretic framework to study such questions~~[17]~~.

Emergence of concept combinations through entanglement

At this point, we need to enlarge our explanation of entanglement so we can usefully apply it to leadership. In physics terms, we can think of a system that contains two particles and that these two particles collide with each other. After collision each particle is separated from the other. However, the pure state of the system remains a Hilbert space superposition of wavefunctions of individual particles, that is, subsystems. In this scenario, if observers, Alice and Bob, measure any property of individual subsystems, for example, the direction of spin of the particles, even when the particles sit vast distances apart, then as soon as one measurement is taken, say by Alice on her particle, the result of the measurement on Bob's particle is already determined. It is important to note that measurement of each subsystem is random. Thus, for both for Alice and Bob the probability of observing upwards spin or downwards spin (if we assume that there can be only two orientations of the spin of particles) is 50%, and, further, it is not possible to know the spin direction before measuring. Given these probability conditions, subsystems (that is, the individual particles) are in a random or mixed state.

Entanglement is certainly 'non-classical' since classical correlations (say between Alice's and Bob's systems) can never account for the instantaneous and unobservable 'communication' between the two particles~~[18]~~. We emphasize here that entanglement actually does not mean any instantaneous travelling of signals for communication, which would certainly mean moving faster than the speed of light, but neither is it the same as classical correlation. As strange as this communication seems, it is, nevertheless, easily demonstrable in experiments and, indeed, is used for quantum computing, atomic clocks, MRI scanners, and GPS navigation systems. Even though the communication in entanglement is unobservable, it is highly accurate, and it can be understood by using information theory, which makes the link between quantum theory and social science research possible. John Von Neuman (2018), one of the founding fathers of quantum theory, proposed the Neuman entropy concept: which holds that if the state of a system is denoted by the density matrix ρ, that can be a pure state or a mixed one, then the entropy measure is ρLNρ, where LN is the natural logarithm.

Aerts et al. (2018) argue that we need to consider the system (Alice + Bob's in this case) as one entity, and that any measurement of the systems is a measurement of the whole system. In the appendix we have provided a simple mathematical description of entangled states. To better understand this, we draw on quantum cognition research. The Brussels group (Aerts et al., 2013) are pioneers in quantum cognition entanglement of combinations of concepts/ideas. They argue that potentiality and contextuality in cognition is analogous to a quantum system. In quantum cognition, experimental context interacts with the system to influence the result. Social actor entanglement at its simplest is a system composed of, say, Alice and Bob, who are each sub-systems and as sub-systems act as agents who take sub-measurements; that is, they each make decisions or evaluations. At this simple level of explanation, decisions made by Alice or Bob are random. This randomness is because before Alice does her measurement the outcome is uncertain and the same goes for Bob, but as

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soon as one of them makes a decision (a measurement/observation) and, thus, obtains a result, the result of the other (Alice's) sub system is fully determined because they are entangled systems, including sub-systems and therefore they mimic each other's changes instantly even if they do not know what each other has decided. For example, if we have a superposition of possibilities, say that an asset price can be up or down, this belief state is a superposition of up and down beliefs and is a pure state because no outcome is yet known. However, once measured (that is, someone decides if it is up or down) the superposition collapses to a final state (an actual belief). Furthermore, pure states become entangled with the environment of different people with different beliefs and different information, they become mixed states, and are entangled. Fake news on Facebook is a good example of an information environment, which may influence the pure belief states of agents who read it and use it for final decisionmaking, creating large scale shifts in behavior. The stock market is another example. When large numbers of people decide to sell shares based on similar but new beliefs, share prices might plummet quickly.[19]

The standard description in physics is, as we mention above, that measurements must be treated as joint measurements over a subsystem, and added to that, the whole system is always at a pure state, whereas the subsystems are at a mixed state (Susskind & Friedman, 2014). An important implication of this is that we have full information about the system as a whole. In quantum theory, full information means that all the information is available or, more accurately, is accessible for observation even if it has not been observed yet. Going even further, we can think of an ensemble of many pure states. For example, in decisionmaking experiments, when many pairs of decisionmakers are performing the same set of choice-makings. Thus, pure state systems are pure in the sense that they are uncollapsed, or unobserved (yet), and have not been converted into a semantic entity or idea or data point. They are, as it were, 'unsullied' by observation.

Sub- or mixed systems, however, are random and unknown because observation has created semantic entities that are unstable. Subjective interpretations and interpersonal communication introduce interpretive errors and idiosyncratic meanings that are the foundation of instability. The Chinese whispers game is an example of this randomness and instability. It is important, though, that Aerts et al. (2018) have demonstrated that there can be two levels of entanglements in decisionmaking; (1) between-states entanglement and (2) between-measurements entanglement. Aerts et al. (2018) suggested that either the belief states of the decisionmakers can become entangled or the belief states of the agents can become entangled with the measurement process, which is analogical to the workings of quantum physics, where contextuality of experiments directly influences the outcomes of measurements.

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[19] In finance for example, there is a wide literature on soft and hard information: soft being Facebook like environment which is less verifiable and hard being Balance sheet like which is more readily verifiable.

Interestingly, there are three main mathematical conditions that have to be violated to demonstrate contextuality and entanglement. Most importantly, each of these conditions are readily violated in the social world. Those three laws are the:

1. Law of separability: explains whether the probabilities of different events happening can be expressed as products of individual probabilities.~~?~~ This is the standard mutually independent event test. The violations in this test point to a correlation between events or in this case choices made by respondents.
2. Law of marginal probability: is a mathematical extension of the first law. In this law, we can say that A has two values A1, A2, then for a context B $P(A|B)=P(A1|B)+P(A2|B)$. However, the total probability formula in quantum theory is fundamentally different from this expression, since it contains interference terms. We have a detailed note in the appendix about the emergence of extra additive interference terms in the formula.
3. Clauser, Horne, Shimony, Holt (CHSH) inequalities: is perhaps the most used and important inequality type for demonstrating deeper correlations between events than cannot be predicted by classical probability theory. Following Aerts et al. (2018)~~,~~ we can have two dichotomous variables, A and B, such that A can have values (A1, A2) or (A1', A2') when A is changed to A', B can have values over (B1,B2) or (B1',B2'). Based on the classical probability theory it can be shown (): $-2<CHSH<+2$, where $CHSH= E(A,B)-E(A,B')+E(A',B)+E(A',B')$, where E(.) are the expectations or joint probability values.

Fundamentally, if the CHSH measure in an experiment violates the range then the events are correlated with each other in a more profound way than in classical probability theory, and it violates the predictions of Kolmogorovian measure theory.

If all such inequalities are violated in cognition experiments (as shown by Aerts et al., 2018), then, there is high degree of entanglement in decisionmaking. Which means that there is entanglement not only among states/ events but also among measures. Aerts et al. (2018) have used the same approach for decisionmaking experiments, where A's and B's are concepts; for example, placing animals and acts in two different sets. In one set we have animal names and in the other we have acts which may or may not be associated with the animals. The A's and B's will, therefore, have different pairs of values. Frequencies or E(.)s are computed based on the frequencies of joint choices made for each A,B pair by respondents, and then CHSH is tested. Violation is always detected.

Emergent cognitive state

The above empirical and theoretical considerations however also indicate that human mind is more complex than either quantum logical or classical logical. There is no clear ground to assume that cognition would always be represented by a full quantum formalism. Hence it would be better (Geneva-Brussels approach) to conceive of a complex description of mind, a combination of quantum logical and classical logical states.

Mathematically, such description can be provided by 'Fock' space representation, which is a more general state-space with direct sums and tensorial products of individual Hilbert spaces. More specifically;

$$|AB> = m\, e^{i\varphi}(|A>+|B>)/2^{1/2}+(1-m^2)^{1/2}e^{i\varphi'}|C>$$

Where $|A>$ is in H
$|B>$ is in H , and $|C>$ is in $H \otimes H$, m [0,1].

the second term is a tensor product representation which can be thought as a product state which might be used for satisfying classical logic inequalities, where as the first term is a superposition representation which might be used for representing deviations from classical logical inequalities, inequalities refer to the basic set theoretic probability rules. Hence the general state |AB> is rather a superposition of two: equivalently a Fock space representation. Human mind is more closely like this: emergent.

Modeling Wise Leader Interaction with Context

We need leaders to act wisely and leadership research needs to account for wisdom (Mumford, 2011). wisdom, however, is one the most challenging social science constructs to research (Sternberg, 2003) but it is ideally suited to a QM research design. Based on the above exploration we further summarize some specific directions in which such modelling might prove productive for leadership research where the goal is to develop excellence, that is, wisdom, in our leaders for the benefit of the planet. To do this, we draw on the multi-level social practice wisdom (SPW) framework (McKenna et al., 2009; Oktaviani et al., 2015; Rooney, 2013; Zhu, Rooney, & Phillips, 2016), which has been used in the context of leadership research and translates well to QM research. Social Practice Wisdom understands wisdom as excellence in social practice the depends on integration of (1) Qualities of mind, (2) agile, transcendent and reflexive reasoning, and (3) ethical purpose and virtuosity in one's everyday life, including leadership. This complex and indeterminate integration produces (4) praxis (wise practice) when successful and it (5) creates short and long-term positive change for the conditions of life on our planet. We now briefly consider each of these five theoretical elements in turn.

1. Qualities of mind and consciousness: An aware, equanimous, compassionate, humble, and actively open mind with an integrated habitus of dispositions that drive insightful and virtuous action. This involves mindfulness, empathy, non-attachment (distancing), acceptance, and self-awareness to understand uncertainty and the relativities of life, including conflicting values, identities, cultures and politics, as well as imperfect

knowledge. This is a complex ontological constellation and there is no formula for predicting how to integrate these factors to produce wisdom in any given situation. The cognitive, affective and cultural context is clearly vastly complex and it seems misguided to treat wisdom as a radically parsimonious version of these factors. Quantum modeling is the best opportunity we have to embrace this complexity quantitatively.

2. Agile, transcendent and reflexive reasoning: Reflexively integrating knowledge, including aesthetic knowledge (direct, embodied, sensory, non-rational knowing and conceptual knowing), transcendence (e.g. creativity, foresight, intuition, trans-conceptuality [non- linguistic knowing]), different perspectives, and clear insight to adroitly deliberate and judge to assist transformative understandings of a situation despite uncertainty and ambiguity. The creative, meaning-making, learning, deciding, and judgmental aspects of wisdom are clearly non-trivial. Given the breadth of mental qualities that wisdom needs to be able to draw on, QM, by understanding them as existing in multidimensional Hilbert space and becoming entangled in the act of reasoning can begin to unpick the hitherto very difficult to access empirically mental dynamics of wisdom in leaders.

3. Ethical purpose and virtuosity: This includes virtues, ethical competence, and the ability to understand and act positively in response to people's emotional, social and material needs. Furthermore, it entails ego transcendence and virtuous alignment of values with social behavior; and insight into the human condition and shifting social relations to find the right and virtuous thing to do at the right time. Self-transcendence and working to a higher purpose and critical in the ethicality of wisdom, this is by definition about phenomenological entanglement, through shared consciousness, communication of ideals, and culture. The complexity of this kind of correlation of entangled beliefs is challenging empirically and analytically but QM presents as a good candidate for moving the wisdom and leadership research effort forward by meeting the complexity without a reductive epistemology and excludes empathy, for example.

These first three qualities and abilities recursively interact with each other as a habitus (or system of dispositions) to create the conative impulse for an embodied wise praxis that leads to excellent outcomes that improve the conditions of life. The degrees of freedom that necessarily apply to this three-part integration in the attempt to be a wise social practitioner, a leader, is vitally important to understand, yet we have not sufficiently developed the methods to do this in leadership research.

4. Embodiment and Praxis (or mastering wise action): Drawing from one's habitus of dispositions to creatively, responsively, and decisively embody and enact wise performative skills in a situation. Wise performance draws on experience and understanding and is based on judgements that are executed and communicated in a timely and aesthetic way. This involves sensing and knowing why, how, and when to adapt to the surroundings and why, how, and when to change them, and how to

astutely make necessary trade-offs. The very idea of habitus makes it clear how important context is to social practices like leadership. Habitus speaks directly to the quantum axiom that the events experienced as reality are emergent properties of contexts.

5. Outcomes that improve the conditions of life: This involves galvanizing, purposeful leadership and artful communication to effect virtuous change with exceptional outcomes. Creating positive cultures and sustainable communities are central to this. Ultimately, we need leadership to be a significant driving force to creating improved conditions of life. But as researchers, we might be humble enough to say that we still need to do better in assisting this process. Indeed, research, and, therefore, researchers, can understand themselves as leaders. We argue that part of that leadership role we can play is the relentless pursuit of new and more suitable approaches to research that will enable us to be those leaders.

Quantum field modeling of decisionmaking enables analysis of leaders' interactions with the information environment (Bagarello, 2015; Khrennikova & Patra, 2019). Quantum field theory is useful for describing instantaneous interactions of a decisionmaker with their information environment. In physics, quantum field theory (QFT) integrates special relativity theory and non-relativistic quantum mechanics. For this article, QFT is of interest as a mathematical toolset that focuses on creation and destruction operators and their commutation rules.

Although Bayesian learning models are used to research adaptive decisionmaking, they have ~~many~~ limitations (Haven et al., 2017). An important advantage for quantum field theory is that it can accommodate the large number of degrees of freedom in the information environment (which in our context includes many different categories of information: hard information which is verifiable, soft in formation which is less verifiable, media, noise, etc.), and then describe how individual decisionmaker's belief states interact with the environment. Technically, we can imagine a decisionmaker's initial belief state as a pure state that is a simple superposition of a few possibilities. However, this state is irreversibly correlated/ entangled with the information environment as soon as the observer queries it and makes semantic sense of it. Over time, an updated steady state evolves as beliefs are modified through learning. Learning is modelled via a decoherence mechanism that collapses a pure superposed state to a mixed state by building an operational theory based on quantum field theory tools.

Any pure decisionmaking state has to interact with the environment. Hence the evolution of the pure/ isolated state $\rho 0$ will, in general, be non-unitary: $\rho(t) = U(t)\rho 0$, where $U(t) = \exp(-itL)$ (1) with L being the generator of GKSL[20] equation (a Lindblad

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[20] These equations are known as Master equations in quantum theory, which describe generally how a systems state evolves over time with interactions and with the information environment embedded in the equation's parameters.

equation that describes the non-unitary evolution of the system). Equation (1) describes the adaptation of the isolated system to the surrounding environment called the reservoir or R, with large degrees of freedom (indicated by a parameter K). Hence the direct way to study the dynamics is to set up the L function and use Heisenberg dynamics: d/dt(ρ(t) = ([H,ρ] +L}. This formulation is interesting since L, the so-called super operator, which maps density matrix to density matrix, contains environmental d.of.

If we consider the pure state of the decisionmaker as S, then the separable S+R (that is, sub-system or our decisionmaker (S) plus the reservoir (R)) state space has a unitary evolution as a whole, which is provided by the Hamiltonian of the compound state. However, the interaction between S and R induces entanglement which makes the compound state non-separable. Hence, the state of the subsystem S becomes mixed. Furthermore, to obtain information for S we then need to take the partial trace for all degrees of freedom of R. We then study the dynamics of the subsystem R with the non-unitary evolution (1).

If Alice is a leader, then Alice's pure belief state is captured by ρ0 = Iϕ><ϕI, where Iϕ> and is the pure, uncertain state described as superposition of I0> and I1>, where I0> can be a no response to the dichotomous question (or the observable here, say A) and I1> the response, yes. The reservoir, or R also comprise of many agents like Alice, who are faced with the same A question, which introduces a large number of dichotomous degrees of freedom. Hence in the state space of Alice, a 2D complex Hilbert space, I0> and I1> forms the orthonormal basis vectors.

The Reservoir, or R, also comprise of many agents like Alice, who are faced with the same A question, hence also comprises of dichotomous degrees of freedom. Hence, in the state space of Alice, a 2D complex Hilbert space, I0> and I1> forms the orthonormal basis vectors.

Alice's decisionmaking process (subsystem S), or the R is described in terms of creation-annihilation operators, a, and a* for Alice and b(K), and b*(K) for the bath/ R. K being the degrees of freedom of the reservoir. The anticommutation algebra for the operators (Fermionic operators as in QFT) is given by {a,b}=0, $a^2=b^2=0$. Where {a,b}=ab+ba.

The operations of a, a* on I0> and I1> is standard: a*I0> =I1>, a*I1>=I0>, and so on. Again the initial conditions are: a*(0) =a*, a(0) = a.

Hence, we come up with the representation of A, or the question posed to the agents as a number operator: N= aa*, where the eigen values of N are 0 and 1, authors ~~(op cit)~~ categorize such an operator as decision operator. Where the average of the decision operator N(average)= <a*(t)a(t)I> (tensor product with I). This average is with respect to some initial states of the compound system (RUS).

Since the agent's belief state is entangled with R(K), the density matrix $\rho(t)$ can be obtained through partial trace over environmental degrees of freedom: $TRKR(t)$, hence, $<N> = TR\rho(t)N$. For dichotomous observable A, this average coincides with the probability $A=1$, hence we can study the dynamics of this probability.

Social interaction Dynamics

A typical closed system will evolve according to the Shrödinger mechanics $\phi(t) = \exp(-iHt)*\phi(0)$. However, here we have interaction between S and R(K), where there is a large degree of freedom (K). We assume here that S (Alice) will interact with immediate next subsystem S'. Hence for S U S', the Hamiltonian of the system $H = a\,NS + b\,NS' + c(a^*b + b^*a)$, where the operators NS and NS' are decision operators for S and S' respectively, and a,a*, and b,b* have usual anticommutation properties. In this case the bracketed term describes the interaction which in a very simple case describes if NS increases by one unit and NS' decreases by one unit. Since N+N' commutes with H, it is an integral of motion, or is conserved. In such cases, the law of unitary evolution based on the Heisenberg model is applicable. However, we should be careful, since unitary evolution where the norm of the state vector at the start may remain~~s~~ conserved, and this may not be the case in decision theory models (Bagarello et al., 2018). Hence, in some models we may also need to consider non unitary evolutions.

Implications for leadership and wisdom research

We believe that three elements of quantum theory are useful for explaining complex features of the practice of leadership. These are (1) adequately accounting for context by using a quantum probability framework (Pothos & Busemeyer, 2013), (2) Quantum Bayesianism (QBism) as a general framework for decisionmaking and cognition, and (3) quantum field theory and decoherence theory-based (Bagarello, Haven, & Khrennikov, 2017) frameworks for decisionmaking and cognition in a complex interactive context with large degrees of freedom.

An instructive research example is the application of quantum field theoretic formulations to asset markets (Bagarello & Haven, 2014; Patra, 2019). These studies focus on modelling interactions between traders based on operator formalism in quantum field theory. For example, in the earlier two-agent model (Alice and Bob), if we introduce the interaction between the agents and the information environment, the so-called reservoir (which can be considered as a vast reservoir comprising of many degrees of freedom, comprised of hard and soft information), then a new model of decisionmaking can be formulated. In this case, agents may start with initial pure states of beliefs, however, the decoherence theory of quantum mechanics can measure when they interact with the information environment as the pure state 'decoheres' to become a mixed states;~~,~~ that is, to become an actual belief about something in particular at a particular point of time. Do deal with this analytically, we can formulate the Hamiltonian value of the system because it is comprised of different creation and destruction operators and their commutation relationships. Acknowledging this complexity means also acknowledging that there are different conserved quantities represented by number operators. For example, in a restricted model the total number of shares traded in a market can be conserved. Finally, the time evolution of such operators would also provide us with time evolution equations for the market as a whole, which can now be computed.

Researchers (Busemeyer & Wang, 2018) have recently developed a procedure based on multidimensional Hilbert space modelling which predicts: (1) the degree of contextuality in a data set and (2) given true contextuality in the data, it describes, or predicts, how outcomes were obtained. Future leadership research can use QM to:

1. model contextuality in leadership to understand how agents form beliefs about how to be a wiser leader.
2. model multidimensional Hilbert Space Modelling (HSM) as a predictive model that can predict wise leadership.
3. apply quantum modelling for describing contextual dynamics in wise leadership.

Specific QM methodological advances that would assist in advancing leadership theory that will help us understand wise leadership as a practice. Potential methods include:

1. using Dzafarov and Kujala (2016), and Busemeyer and Wang (2018) framework based on general joint distribution of random variables, expressed in terms of inequalities (Bell inequalities or CHSH inequalities), and have used multidimension Hilbert space modelling (HSM) to predict decisionmaking outcomes (we have provided basic outlines of such frame work in the appendix). This method is directly applicable to leadership research. Most HSM models use four random variables. Such variables are dichotomous values. One variable could be for wise leadership, the other variables could be an important variables that may or may not be compatible with wisdom.
2. Using the HSM model could also compute the conditional belief state of a wise leader given the values of other variables.
3. Using compatibility and order effect analysis in Quantum modelling. If questions related to specific variables are represented by self-adjoint operators, then the final state of an actor will alter if the order/ sequence of questions is altered. Symbolically, [X,Y] not = [Y,X], there is a developed mathematical description of this, non-commutation. Where [X,Y] = XY-YX, where X and Y are random variables or observables which are provided operator representations, such operators may be Hermitian or non-Hermitian (please see appendix). An extension to leadership would

be how a leader's judgement on a specific matter changes once contexts X, Y are placed in different orders.

The founders of the Quantum Bayesian school (Caves et al., 2002) interpret their theory as inherently a decisionmaking theory. However, because this school subscribes to a subjective and personalist view of quantum measurement, they imply that decisionmakers are knowledgeable about the underlying (quantum) decision rules. Since quantum Bayesian interpretations of quantum theory are personalist (subjective), rather than objective, and acknowledge the role of knowledge, leadership is a natural field for its extension. We suggest that QBism can offer insights for leadership, generally, and wise leadership in particular. Wisdom is an inherent human quality present in truly excellent leaders when they have stood out in ambiguous and difficult contexts (Mahatma Gandhi, Nelson Mandel~~l~~a, etc.).

Wise Leaders as entangled actors

Quantum field-inspired modeling is used for describing dynamic cognition between agents entangled in a given information environment. Thus, because organizations, leaders and wisdom are quantum-like systems, research can assume that those systems "predispose the possibility space for different configurations of interwoven tensions, [and] their actual enactment depends on the specific socio-material context" (Hahn & Knight, in press). Leadership contexts are clearly a fertile ground for applying such a model. Leadership judgment and communication are carried out in a complex information environment where a leader's initial isolated belief state, can be modelled as a pure state superposition. This approach models how leaders are entangled with the information environment, with many, or even infinite, degrees of freedom. In the entanglement process, isolated states lose their 'purity'; that is, they 'decohere' as they become entangled with the overall belief environment and develop particular understandings that lead to decisions to act in particular ways. Again, we are theorizing the belief states of the actors only. It would be intriguing to analyze the role of a leader and follower entanglement in a belief state world. There is a tradition of leader-follower game theory models, which produce different Nash equilibria compared to simultaneous move games. However, in an entangled belief world, standard game theory solutions might not work. We emphasize here that entanglement means losing the purity of one own state and becoming correlated with the surrounding environment.

Applications of quantum decision theory to game theory are at a nascent stage (Yukalov & Sornette, 2011). However, promising research has been done using the strategy profiles of players acting in adaptive information environments. A strategy profile is the set of strategy choices that players have, as in standard game theory, however, quantum game theory (Piotrowski & Sładkowski, 2003) greatly expands the available choices to more closely represent reality and this changes the Nash Equilibrium solutions significantly. Interestingly, researchers (Piotrowski & Sładkowski, 2003) have shown that if ‘quantum strategies’ are accessible for the players, games can find equilibrium solutions much earlier than predicted in standard games. Using quantum game theory, we may observe that quantum strategies are responses made by actors (corresponding to best response curve in standard game theory) that are analogous to operations allowed in quantum information theory. That is, intelligent people entangled in an information environment and who are subject to the influences of order effects, learning, etc.

Adaptive information environments assumes that games are played in different contexts and that and intelligent players do not behave according to standard Nash Equilibrium models when uncertainty and ambiguity pertain. Under such conditions, players actively respond to contexts and, in particular, to their information environment. In other words, they have to explore, learn, communicate, and adapt to find a course of action. Leadership communication and cognitive process is a most productive ground for the extension of such models.

Final Comments

We return now to the very early point made in this article, that quantum-like modelling is different not identical to standard quantum physics, but provide more information about why?

Research (Baaquie, 1997, 2018) shows that there are significant differences between Quantum-like modelling in social science and standard quantum physics. Though the current article is not designed to provide a complete discussion of this literature, we nevertheless want to finish our discussion by pointing out the most salient differences.

- Quantum like modelling often requires non-Hermitian Hamiltonians, which means the Hamiltonians describe systems that are not equal to their complex-conjugate transposes. In standard quantum physics this is not always required, since a Hermitian Hamiltonian guarantees real Eigen values. When dealing with non-Hermitian Hamiltonians, we need to adopt nontrivial techniques to describe the dynamics of systems (Baaquie, 2018). It is still not entirely clear whether a general theory can be established here, which warrants further research.
- For the most part, it is decisionmaking models that warrant time dependent Hamiltonian operators, which give rise to violations of probability conservations.

- Underlying state space in social systems may not be the same as standard finite or infinite dimensional Hilbert space, but a more complex Fock space or even a time dependent state space.
- Entanglement observed in decisionmaking models are more complex than in the physical world.
- Entanglement does not mean any kind of superluminal communication occurs in signaling between subsystems of the composite system, the same is also true in case of entanglement for human cognitive experiments. Dzhafarov et al. (Dzhafarov & Kujala, 2016) have demonstrated that many cognitive experiments have claimed entanglement might be undermined if some kind of signaling is present between subsystems, since in the presence of signaling CHSH inequalities can be violated but that in such cases it is not true 'contextuality', rather it may be the influence of one subsystem's result by another. Hence, recently scientists () have claimed that such signaling effects have to be controlled for if CHSH inequalities are violated because if the signaling measures are introduced and subtracted from LHS of the CHSH inequalities we can be sure of true contextuality. This caveat is important since in case of organizational or leadership decisionmaking processes there may be ample of opportunities for such signaling, for example, group members in a decisionmaking process influencing each other's results via hidden signaling.

These differences signify that the QM paradigm is unique and, for example, can address non-linearity, non-ergodicity, chaotic dynamical systems, and of analytically challenging dynamics in social systems[21]. We see much scope for extending QDT models to leadership cognition, communication and judgment to transform leadership theory[22].

To conclude, then, we would like to observe that there is a growing interest in extending the QM framework in organizational behavior and management research, however, leadership can also provide a rich and promising ground for empirically testing some of the central techniques of QM. The underlying ontological framework that QM uses, including its focus on context, ambiguity, and entanglement offer the promise of new kinds of research designs that enable researchers to redouble their efforts to fully understand leaders and leadership. We can do this by contributing the foundational knowledge with which to better develop leaders, better predict leader performance, and, ultimately, we hope, to bring much needed wisdom to a global leader cohort; something the world desperately needs given our

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[21] In this regard QDT can also play a fundamental role in complexity theory, which describes society and economy as a complex dynamical system, with deep uncertainty.

[22] These modelling challenges are significant, however, recently models have been devised to tackle such uniqueness in decision making models (Bagarello, 2015 & Khrennikova and Patra, 2019)

uncertain future and complex and rapidly changing social, economic and environmental world.

Appendix 1

Basic concepts in Quantum Mechanics

Quantum physics began when Max Plank and Albert Einstein proposed that energy can only be radiated and absorbed in small units, now called quanta, and that light or electromagnetic radiation are streams of massless particles called photons. The word quantum mechanics was coined in 1920s by Heisenberg, Born, Pauli, Jordan and other eminent scientists. The core structure of Quantum theory was built by 1930s, and since then scientists and philosophers have continued to develop it.

Prior to quantum physics, classical physics was tied to three principles: (1) locality, which demands that there has to be a speed limit to signaling between events in space-time, which is challenged by entanglement, (2) causality, which demands a strict cause and effect relationship in nature, or a strict one directional arrow of time, and (3) realism which demands a subject-object split in (an objective) nature. However, at the quantum level of reality each of these principles is violatable. Such bold new ontological insights changed the course of modern physics by challenging classical assumptions about the nature of the physical universe and even the idea of an objective reality.

To compliment this new interest a new language of mathematics and logic was developed for quantum research by such people as Heisenberg, Shrödinger, Born, and Neuman. Quantum statistics, in the form of Boson and Fermion statistics enables significant research break-throughs by, for example, Satyen Bose and Einstein.

Theoretical advances, most notably by Richard Feynman, gradually reformulated quantum mechanics by, for example, introducing the path integral or sum over histories technique, which opened the door for quantum field theory, quantum electrodynamics, and the ‘standard model’ of particle physics, which remains the most successful model of the universe.

Here we just provide a few definitions of the basic objects in quantum mechanics, in appendix 3 we provide a more detailed account of the mathematical structure of the theory.

Wave function: the description of a quantum state or a quantum system, a complex amplitude, whose modulus squared (square of the absolute value) provides the probability of the system to be found in a specific region. Wave function is described in a superposition of possibilities, or eigen values, until it is measured/observed. Wave function evolves over time in a deterministic manner following an equation of motion, namely Schrödinger’s equation of motion. The wave function lives in a complex normed vector space, named as Hilbert space.

Measurement: wavefunction evolves deterministically, until the experimenter measures a specific property of the system: for example, position, or velocity, or spin. Orthodox views suggest that measurement makes the wave function collapse to one of the eigen values measured/observed in the superposition. However, this process of measurement and collapse is a truly random process and is not dependent on our state of knowledge of the initial conditions of the system. Hence, randomness in quantum theory is ontological rather than epistemological.

More recently, some of the features of quantum reality such as contextuality, entanglement, and observer effects have drawn the attention of social scientists because social systems have important quantum-like features to which the logical and statistical tools of quantum physics can be applied.

Appendix 3

Basic mathematical tools or concepts

We begin with a brief comparison between classical probability theory (CPT) and quantum probability theory (QPT):

The main features of classical probability theory are:

Events are represented by sets, which are subsets of Ὠ

Sample space, sigma algebra, measure (probability)*, are the main features of the related Kolgomorov measure theory.

Boolean logic is the type of compatible logic with CPT, which allows for deductive logic, and basic operations like union and intersection of sets, DeMorgan Laws () of set theory are valid.

Conditional probability: P(a/b) = p(a and b)/p(b); p(b)>0We see conditional probability is a direct consequence of Boolean operations

Based on the Boolean logic the set theory of probability also directs to Bell’s inequalities: P(AandB) +P(B- andC)>/=P(AandC)

The main features of Quantum Probability Theory are:

State space is a complex linear vector space: Hilbert space***; Finite/ infinite D, symbolized as H

H is endowed with a scalar product (positive definite), norm, and an orthonormal basis, non-degenerate

Any state can be visualized as a ray in this space

Superposition principle: which states that a general state can be written as a linear superposition of 'basis states', in information theory language the basis states are |0> or |1>.

Measurement: most of the times projection postulate**

Measurement implies projection onto a specific Eigen sub-space

Probability, updating can be visualized as sequential projections on Eigen subspaces

Non –Boolean logic is compatible with such state space structure, which means violation of commutation and associative properties.

The main features of Non-Boolean Logic are:

Algebra of events is prescribed by quantum logic

Events form an event ring R, possessing two binary operations, addition and conjunction

P(A U B) = P(BUA) (this Boolean logic feature is invariant in Quantum logic).

P{AU(BUC)}= P{(AUB)U(AUC)} (associative, property also holds good)

AUA = A (idempotency)

P(A and B) # P(B and A) (non commutatitivity, incompatible variables)

A and (B UC) # (A and B) U (A and C) (no distributivity)

The fact that distributivity is absent in quantum logic was emphasized by Birkhoff and von Neumann. Suppose there are two events B1 and B2 that, when combined, form unity, B1 ∪ B2 = 1. Moreover, B1 and B2 are such that each of them is orthogonal to a nontrivial event A #0, hence A ∩ B1 = A ∩ B2 = 0. According to this definition, A ∩ (B1 ∪ B2) = A ∩ 1 = A. But if the property of distributivity were true, then one would get (A ∩ B1) ∪ (A ∩ B2) = 0. This implies that A = 0, which contradicts the assumption that A # 0.

The main features of Quantum-like Modeling of Belief States are:

Bruza and Busemeyer (2015): cognitive modelling based on quantum probabilistic frame work, where the main objective is assigning probabilities to events

Space of belief is a finite dimensional Hilbert space H, which is spanned by an appropriate set of basis vectors

Observables are represented by operators (positive operators / Hermitian operators) which need not commute

$[A,B] = AB - BA = 0$

Generally, any initial belief state is represented by density matrix/ operator, outer product of ψ with itself $\rho = |\psi\rangle\langle\psi|$, this is a more effective representation since it captures the ensemble of beliefs

Pure states and mixed states

Mixed states: $\sum w\,|\psi\rangle\langle\psi|$, hence mixed state is an ensemble of pure states with w's as probability weights.

Some properties of ρ: $\rho = (\rho*)T$, for pure states $\rho = \rho^2$, where T stands for transpose operation.

$(\psi, \rho\,\psi)>0$: positivity, Trace $\rho = 1$

Measuring the probability of choosing one of the given alternatives, which is represented by the action of an operator on the initial belief state

While making decision superposition state collapses to one single state (can be captured by the Eigen value equation)

Observables in QPT represented by Hermitian operators:

$A = (A*)^T$

$E(A) = Tr(A\,\rho)$, every time measurement is done one of the Eigenvalues of the A is realized

$A=\sum aP$ spectral decomposition rule: a's are the Eigen values and P's are the respective projectors which projects the initial state to the Eigen subspace with a specific a

Trace formula: $p(ai)= Tr(Pi\,\rho)$

As soon as the measurement is done the state ρ': $Pi\,\rho Pi/ Tr(Pi\,\rho)$

Simultaneously updating of the agents’ belief state

A QUICK REVIEW OF FORMULA FOT TOTAL PROBABILITY / LAW OF TOTAL PROBABILITY (LTP), MODIFIED IN QUANTUM LIKE SET UP

First we see the LTP in classical set theory as below:

P (B and (A or C)) = P(B and A) +P(B and C)

(measure theoretic additivity)

P(B and A) = P(A)P(B|A), and, P(B|A) = P(B and A)/P(A)

Hence it follows:

P(B | (Aor C)) = P(B|P(A or C) =

{P(B|A)*P(A)+P(B|C)*P(C)}/P(A or C)

Hence in particular if P(A or C) =1, then P(B)={P(B|A)*P(A)+P(B|C)*P(C)}, this is the LTP (law of total probability) as we know in familiar CPT(classical probability theory).

But in the QPT (quantum probability theory) additivity does not follow, which means LTP is violated since there are interference terms

To get the modified LTP as in non Kolgomorovian QDT set up we have to go through the concept of positive valued operators (POVM) as below:

A positive operator valued measure (POVM) is a family of positive operators {Mj} such that ∑Mj = I, where I is the unit operator. It is convenient to use the following representation of POVMs:

Mj = V* j Vj,

where Vj : H → H are linear operators. A POVM can be considered as a random observable. Take any set of labels α1,...,αm, e.g., for m = 2,α1 = yes,α2 = no. Then the corresponding observable takes these values (for systems in the state ρ) with the probabilities p(αj) ≡ pρ(αj) = TrρMj = TrVjρV*j.

We are also interested in the post-measurement states. Let the state ρ was given, a generalized observable was measured and the value αj was obtained. Then the output state after this measurement has the form: ρj =VjρV * j /(TrVjρV* j)

Both order effects and interference terms in LTP can be demonstrated using POVM

Consider two generalized observables a and b corresponding to POVMs $Ma = \{V * j\ Vj\}$ and $Mb = \{W* j\ Wj\}$, where $Vj \equiv V(\alpha j)$ and $Wj = W(\beta j)$ correspond to the values αj and βj. If there is given the state ρ the probabilities of observations of values αj and βj have the form:

$pa(\alpha) = Tr\rho Ma(\alpha) = TrV(\alpha)\rho V*(\alpha)$, $p(\beta) = Tr\rho Mb(\beta) = TrW(\beta)\rho W*(\beta)$.

Now we consider two consecutive measurements: first the a-measurement and then the b-measurement. If in the first measurement the value $a = \alpha$ was obtained, then the initial state ρ was transformed into the state

$\rho a(\alpha) = V(\alpha)\rho V*(\alpha) / (TrV(\alpha)\rho V*(\alpha))$

For the consecutive b-measurement, the probability to obtain the value $b = \beta$ is given by

$p(\beta|\alpha) = Tr\rho a(\alpha)Mb(\beta) =$

$TrW(\beta)V(\alpha)\rho V*(\alpha)W*(\beta)/(TrV(\alpha)\rho V*(\alpha))$

This is the conditional probability to obtain the result $b = \beta$ under the condition of the result $a = \alpha$. We set $p(\alpha,\beta) = pa(\alpha)p(\beta|\alpha)$.

Now since operators need not commute $p(\alpha,\beta) = p(\beta, \alpha)$

We recall that, for two classical random variables a and b which can be represented in the Kolmogorov measure-theoretic approach, the formula of total probability (FTP) has the form $pb(\beta) = \sum pa(\alpha)p(\beta|\alpha)$.

Further we restrict our consideration to the case of dichotomous variables, $\alpha = \alpha1,\alpha2$ and $\beta = \beta1,\beta2$.

FTP with the interference term for in general non-pure states given by density operators and generalized quantum observables given by two (dichotomous) PVOMs:

$pb(\beta) = pa(\alpha1)p(\beta|\alpha1) + pa(\alpha2)p(\beta|\alpha2) + 2\lambda\surd\{pa(\alpha1)p(\beta|\alpha1)pa(\alpha2)p(\beta|\alpha2)\}$,

or by using ordered joint probabilities $pb(\beta) = p(\alpha1,\beta) + p(\alpha2,\beta) + 2\lambda\beta\surd p(\alpha1,\beta)p(\alpha2,\beta)$. Here the coefficient of interference λ has the form: $\lambda= Tr\rho\{W*(\beta)V*(\alpha i)V(\alpha i)W(\beta)-V*(\alpha i)W*(\beta)W(\beta)V(\alpha i)\}/ 2\surd pa(\alpha1)p(\beta|\alpha1)pa(\alpha2)p(\beta|\alpha2)$
Introduce the parameters

$\gamma\alpha\beta = Tr\rho W*(\beta)V*(\alpha)V(\alpha)W(\beta) / (Tr\rho V*(\alpha)W*(\beta)W(\beta)V(\alpha)) = p(\beta,\alpha)/p(\alpha,\beta)$

This parameter is equal to the ratio of the ordered joint probabilities of the same outcome, but in the different order, namely, “b then a” or “a then b”. Then, Interference term $\lambda = \frac{1}{2} \{\sqrt{(p(\alpha 1, \beta)/p(\alpha 2,\beta)} *(\gamma\alpha 1\beta -1) + \sqrt{(p(\alpha 2, \beta)/p(\alpha 1,\beta)} *(\gamma\alpha 2\beta -1)$

In principle, this coefficient can be larger than one. Hence, it cannot be represented as $\lambda = \cos\theta$ for some angle ("phase") θ, cf. However, if POVMs Ma and Mb are, in fact, spectral decompositions of Hermitian operators, then the coefficients of interference are always less than one, i.e., one can find phases θ.

One important note is that such phase terms cannot always be expressed in trigonometric terms, Hyperbolic phase terms are also possible, which are typical of results obtained from decision making models (Haven and Khrennikov, 2013).

Entanglement mathematics

As we have seen throughout that quantum theory allows superposition of the basis states to form new states, many of such superpositions, but not all, poses the quality of entangled states. For example, we start with a qubit system (i.e. a system which has only two basis states |0> and |1>, where they may represent up and down states, for example in decision making models they represent belief sets of decision makers as up state or down state related to any future event), now such a system can be written in superpositions of the basis states in a number of ways:

|x> = 1/√2 {|00>+|11>}, this state can be called as an entangled state, since say if these qubits are given to Alice and Bob, and even they are separated light years apart, if Alice measures her system there is always a 50-50 chance of finding a |0> or |1>, however as soon as she discovers that it is determined with 100% probability that Bob has to have |0> in the first case and |1> in the second case.

Hence there is no superluminal communication happening, only that subsystems are in a random state and the system as a whole is in a pure state.

Again, another hallmark of such states is that mathematically they are not separable, in the sense that |x> cannot be written as a sum over tensor products of only |0> or |1>.

Comparatively, separable states are like |y> = 1/√2{|00>+|01>}, in such a case Alice will always with probability 1 measure her subsystem to be in |0> but Bob still will have a 50% chance of |1> or |0>, again |y> can be separated as 1/√2{|0>(|0>+|1>)} which means a tensor product between |0> and the superposition of |0> and |1>.

Measure of degree of entanglement: concurrence measure is a type of measure of degree of entanglement, say a general entangled state is written as: a |00>+b|01>+c|10>+d|11>

Then the state is maximally entangled if |ad-bc|=1, and there is no entanglement if |ad-bc|=0.

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