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PARIKSHIT GHOSH AND NAVEEN THOMAS

O P JINDAL GLOBAL UNIVERSITY
https://jgu.edu.in/

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Parikshit Ghosh* Naveen Joseph Thomas †‡

Abstract

The low level of participation of women in the labour force has emerged as cause of concern from the perspective of gender equality and as an impediment to economic development, in India, over the past few decades. Our study provides a theoretical framework to understand how social structure influences women's labour market choices. We consider household decision making in a two person household with a wife and a husband, and analyze decision making in non-patriarchal and patriarchal social structures. The patriarchal regime is characterized by men having control over women's labour supply decisions and the non-patriarchal regime is characterized as being gender neutral. We find that the patriarchal social structure generates inefficiencies in the labour markets as women are prevented from joining the labour force even if they potentially earn more than their spouses. Adding more structure to the framework and introducing a sector which allows couples to purchase household help from the market, we see that the inefficiency of the patriarchal system persists. Further, the model provides an explanation for the empirical U-shaped labour supply curve of women with respect to their education level. Hence, the study highlights the need for policies that increase the bargaining power of women in patriarchal societies and the ambiguous effect of education on women's labour force participation.

KEYWORDS: Social Structure, Nash Bargaining, Female Labour Force Participation

*Department of Economics, Delhi School of Economics, University of Delhi, pghosh@econdse.org

†Jindal School of Government and Public Policy, O. P. Jindal Global University, naveenthomas@jgu.edu.in

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1 Introduction

The past few decades has seen the emergence of India as the fastest growing major economy in the world (World Bank, 2019). However, the growth story is mired in several contradictions. One of the critically debated contradiction, in recent years, is the low level of Female Labour Force Participation (FLFP). The low level of FLFP is puzzling, since it is accompanied by rapid fertility transition, and broad improvements in women's education attainment and wages, which tends to be supportive of an increase in participation of women in the labour market (Afridi et al., 2017; Fletcher et al., 2017; Kapsos et al., 2014; Klasen and Pieters, 2015). According to the National Sample Survey (NSS) 68^{th} round National Employment and Unemployment Survey, 2011-12, the overall FLFP by usual primary status in the age group 25 and above was 26.32 percent. If we further split the FLFP by urban and rural, we see that the urban FLFP was 19.58 percent and the rural FLFP was 29.22 percent. Comparing India's FLFP with the rest of the world, we see that, according to ILO(2013), India ranks 121 out of 131 countries across the world and one of the lowest in South Asia (Andres et al., 2017).

The low FLFP is a cause of concern for several reasons. The first is that India currently has a large working age population with few dependents. Given that women make up nearly half the working age population, having so few women participating in the labour force has enormous economic implication. Esteve-Volart (2004) in her study shows that, for India, a 10 per cent increase in female-to-male ratio of managers would increase per capita total output by 2 per cent and a 10 percent increase in female-to-male ratio of total workers would increase per capita total output by 8 percent. The second is that women's participation in the labour force also has implication on the extent to which they can benefit from economic growth. This is because employment and earnings are important determinants of women's bargaining power in the family(Anderson and Eswaran, 2009). The third is that there are positive spillovers from women's earned income on child indicators. There is fair evidence that children enjoy better educational outcomes when their mothers earn a wage income(Afridi et al., 2012; Luke and Munshi, 2011). All these factors are extremely critical, because, despite the high rate of economic growth, India's per capita income and Human Development Indicators are very low when compared to the rest of the world and a higher level of FLFP could potentially improve these indicators.

The existing literature tries to explain the phenomenon through socio-economic factors that influence both the demand and supply of women's labour supply. On the demand side, it is argued that economic growth has not translated into higher job creation in general and in particular for women (Fletcher et al., 2017; Kannan and Raveendran, 2012; Klasen and Pieters, 2015; Naidu, 2016). This means that even though women want to work, they do not find suitable jobs.

On the supply side, the predominant argument is that women face social stigma when they engage in paid work. Fletcher et al. (2017) argue that Indian households require women to prioritize household work and may even explicitly constrain work by married women. Further evidence on social strictures on women's participation in paid work is provided by Ghai (2018), who finds that Indian States which have stronger social strictures on women are less likely to have women engaging in paid work, especially at higher levels of education.

This supply side explanation to the low FLFP is the starting point of our study. We provide a theoretical framework to understand the economic rationale behind these strictures on women's participation in the labour force. The basis of the model is the conflict of preference within a two individual household, where the couple pools resources and takes decisions collectively. When it comes to labour supply, economic efficiency would require the higher earning spouse to work full time, irrespective of gender. This provides the households with the highest income and welfare. However, the low FLFP observed in India seems to reflect economic inefficiency. We hence investigate further for a possible explanation.

The answer might lie in the nature of social norms in India. In patriarchal societies, it is within the control of men to prevent their wives from entering the labour force (Derné, 1994). If women are unable to join the labour force early in their working lives, they are forever deprived of the credentials and networks that help an individual remain in the labour force. Thus, not participating in the labour force early in their working lives precludes future access to labour markets. The lack of access to paid work means that they have lower bargaining power within marriage (Anderson and Eswaran, 2009) due to not having any options outside marriage. In patriarchal societies, given the power that husbands have over their wives' labour market decisions, it is conceivable that the labour supply of married women is determined solely to profit their husbands and hence generates a dead-weight loss. The husband has the option of letting the wife work or follow a gender based division of work, where women don't work to prioritize care giving. The first option of letting the wife work has the benefit that the household income and hence welfare is higher given that labour market decisions are based on the overall welfare of the family. The second option of preventing the wife from participating in the labour force comes at the cost of a lower family income and welfare. Despite the loss in overall household income, the husband might still exercise this option because he can potentially gain from it. This is because the loss in the wife's bargaining power due to her not having access to labour markets, even if she breaks away from marriage, means that husband can corner a larger share of the smaller household income and hence raise his welfare over the welfare he would have received by allowing his wife to work.

The model goes further to explain another set of recent empirical findings that link FLFP with the education of women. Klasen and Pieters (2015) provides empirical support for a U-shaped relation between FLFP and the education level of women in urban India. For Rural India, Afridi et al. (2017) find a decline in FLFP with respect to women's education. Our model also helps explain the response of FLFP to husband's wage in a cross section.

The rest of the paper is organized into 5 sections. The second section that follows describes the general framework of household decision making. The third section explains the supply of labour by households when the spouses have no conflict of preference. The fourth section describes the household's labour supply when there is conflict of preferences between spouses and helps explain the low levels of FLFP in patriarchal societies. The fifth section introduces a market for household help and we show that this can generate a U-shaped relation between the labour supply of the wife, and her own education and husband's income. The sixth and the last section summarizes the key findings of this study.

2 Framework of Household Decision Making

In this section we introduce the general framework of household decision making. We analyze the behavior of a representative household which comprises of a husband and a wife, each endowed with a unit of time. They can allocate time between market work $(l_i, i = w, h)$ and household work$(t_i, i = w, h)$. Further, the wife and husband earn market wages αw and w per unit of time spent on market work, respectively and are determined exogenously. Here, α denotes the relative wage of wife with respect to the husband's wage w. In marriage, the couple pools resources when making decisions for the household.

Each spouse cares about two household goods, a wife specific and husband specific private good, denoted by x_w and x_h respectively. They also care about a household public good that is produced using time allocated to household work. Cobb-Douglas[1] utility functions that capture these preferences are given by:

$$u_h = (x_h)^{\sigma}(x_w)^{1-\sigma}T^{\beta} \tag{1}$$

$$u_w = (x_h)^{1-\sigma}(x_w)^{\sigma}T^{\beta} \tag{2}$$

Here, $\sigma = \left\{\frac{1}{2}, 1\right\}$ and $\beta \in (0, 1)$ are parameters that represent the importance of private consumption and household time to each spouse. Further, the household time allocation T is $t_i, i = w, h$, when they are unmarried and $t_w + t_h$ when married. We now analyze the work choices of the spouses in the household assuming identical preferences.

3 Household Decision Making without Conflict of Preferences

In this section we assume that wife and husband have identical preferences with $\sigma = \frac{1}{2}$. Hence,

$$u_h = (x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta} \tag{3}$$

$$u_w = (x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta} \tag{4}$$

This gives us a model of household decision making were the spouses have no conflict of preferences and hence optimize their common utility functions subject to their budget constraint. Further, we assume that household time is not marketed and has no close substitute. The household's optimization problem is given by:

$$\max_{x_w, x_h, t_w, t_h} (x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta}$$

subject to :

$$x_w + x_h = (1 - t_w)\alpha w + (1 - t_h)w$$

$$T = t_w + t_h$$

[1] Quasi-linear utility functions instead Cobb-Douglas utility functions yield similar results.

The following proposition characterizes the households decision making problem in the absence of conflict in preferences between spouses.

Proposition 1. *When there is no conflict of preferences between the wife and husband, labour supply is determined by considerations of efficiency. The spouse who earns more will always work full time irrespective of gender. The other spouse will either do full time domestic work, or split her/his time between market and domestic work. The labour supply of the wife (l_w) and husband (l_h) as a function of the wife's relative wage α is as follows:*

$$l_w = \begin{cases} 0 & if \ \alpha \in (0, \beta] \\ \frac{1-\frac{\beta}{\alpha}}{\beta+1} & if \ \alpha \in (\beta, 1) \\ 1 & if \ \alpha \in [1, \infty) \end{cases}$$

$$l_h = \begin{cases} 1 & if \ \alpha \in (0, 1) \\ \frac{1-\alpha\beta}{1+\beta} & if \ \alpha \in \left[1, \frac{1}{\beta}\right) \\ 0 & if \ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases}$$

The graphical representation of each spouse's labour supply, under collective decision making without conflict of preferences, is shown in figures 1 and 2. The proof for this proposition is provided in Mathematical Appendix A.

When the husband and wife decide on the labour supply, if the wife's relative wage is extremely low (i.e. $\alpha \leq \beta$) the husband works full time to earn the household the maximum income and the wife does household work full time. The wife does not enter the labour force at these low wages because the gain in income by a marginal increase in her labour supply does not compensate the loss in welfare due to the the lower supply of time to household work. As the wife's relative wage rises about β the husband continues to work full time but given the higher opportunity cost of the wife staying out of the labour force, she enters the labour force but does not work full time. The tables turn when the wife's wage rises above the husband's wage. The wife works full time and the husband withdraws to the household sphere though still partially supplying labour to the market. However, when the wife's wage is very high (i.e. $\geq \frac{1}{\beta}$) the husband withdraws entirely from the labour force as his labour market engagement does not compensate the loss in welfare due lower time spent on household work.

The critical take-away here is that the allocation of time by the spouses between household work and labour market is gender neutral.

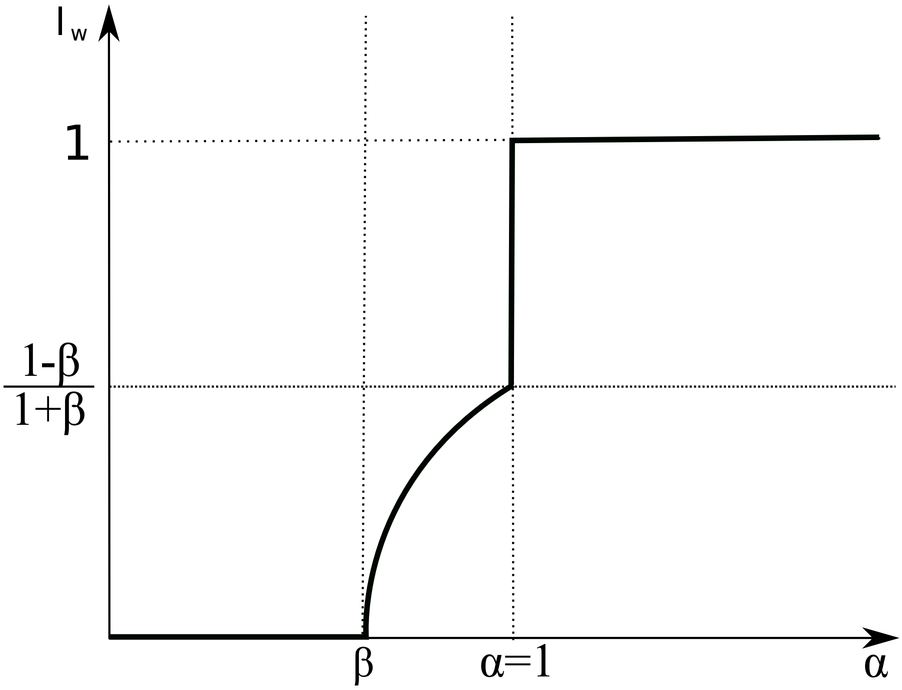

Figure 1: The wife's supply of market labour (l_w) as a function of her relative wage α under collective decision making without conflict of preferences.

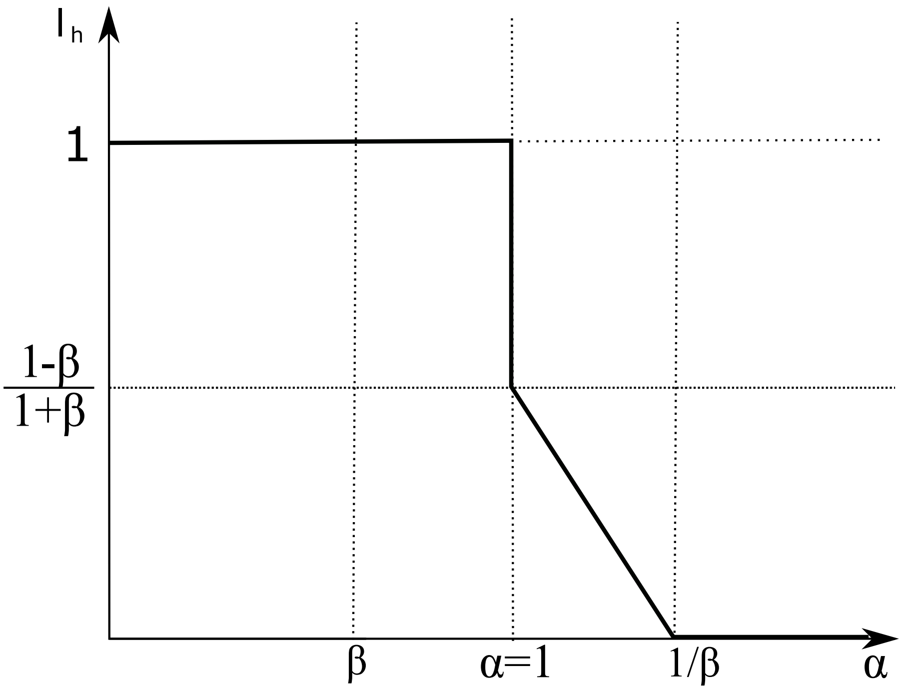

Figure 2: The husband's supply of market labour (l_h) as a function of the wife's relative wage (α) under collective decision making without conflict of preferences.

4 Household Decision Making with Conflict of Preferences

We introduce the idea of conflict of preferences by assuming that the each spouse is individualistic in the sense that the husband doesn't care about the wife's private consumption and vice versa or we assume $\sigma = 1$. The utility functions describing their preferences are now as follows:

$$u_h = x_h \cdot t^{\beta}$$
$$u_w = x_w \cdot t^{\beta}$$

We also define two regimes of household decisions making. A non-patriarchal regime and a patriarchal regime.

In the non-patriarchal regime, the couple pools their time endowment, and jointly decides private consumption of each spouse and their time allocations to household work and market work. The household decision making is formulated as a bargaining problem which is solved using the Nash Bargaining solution. The household's problem is as follows:

$$\max_{x_w, x_h, T} N_N = \left[x_h \cdot T^{\beta} - R_h\right] \left[x_w \cdot T^{\beta} - R_w\right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1 - t_h)w + (1 - t_w)\alpha w \\ T &= t_w + t_h \end{aligned}$$

Here, R_w and R_h are the wife's and husband's threat points. The threat points of the spouses is their indirect utility outside marriage and it forms the basis of their bargaining power within the marriage.

In the patriarchal regime, the husband has the choice of preventing the wife from participating in the labour-force (Derné, 1994). If the wife is prevented from joining the labour force, time allocation is gender specific with the wife doing household work full time and the husband engaging in the labour-force while supplying any residual household time. Household decision making can now be thought of as a two stage decision making problem. The first stage involves the husband choosing between letting the wife work and confining her to the household sphere. In the second stage, once the decision of letting the wife join the labour force has been made, the household decision making problem is again formulated as a bargaining problem where the couple decides their private consumption and time allocation. The bargaining problem when the wife is allowed to work is the same as in non-patriarchal regime. However, if the wife is prevented from joining the labour force, the household decision making problem is again posed as bargaining problem to solve for the couple's private consumption and the husband's time allocation. When the husband restricts the wife to the household sphere, she looses all her bargaining power since she can never enter the labour force even as a single person household, we will discuss the reasons for this when we solve the household's problem. Thus, her threat point goes to zero as she has no outside option in marriage. The household's objective function is hence as follows:

$$\max_{x_w, x_h, T} N_P = \left[x_h \cdot T^{\beta} - R_h\right] \left[x_w \cdot T^{\beta}\right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1 - t_h)w \\ T &= 1 + t_h \end{aligned}$$

Here, R_h is the husband's threat point and the wife's threat point $R_w = 0$. The two stage decision making is solved by backward induction.

We now proceed to analyze the couple's labour market decisions in each of these regimes.

4.1 The Non-Patriarchal Regime

We start by identifying the couple's threat points which, in marriage, are their indirect utilities when single. The couple's indirect utilities when single are obtained from the following decision making problems:

$$\max_{x_h, t_h} x_h \cdot t_h^\beta \quad s.t. \;\; x_h = (1 - t_h)w$$

$$\max_{x_w, t_w} x_w \cdot t_w^\beta \quad s.t. \;\; x_w = (1 - t_w)\alpha w$$

Setting up the Lagrangian for the husband's decision making problem, we have:

$$L = x_h \cdot t_h^\beta + \lambda[(1 - t_h)w - x_h]$$

The first order conditions (FOCs) are as follows:

$$\begin{aligned} \frac{\partial L}{\partial x_h} &= t_h^\beta - \lambda = 0 \\ \frac{\partial L}{\partial t_h} &= \beta x_h t_h^{1-\beta} - \lambda w = 0 \\ \frac{\partial L}{\partial \lambda} &= (1 - t_h)w - x_h = 0 \end{aligned}$$

Solving the FOCs simultaneously we find:

$$x_h = \frac{w}{1+\beta}$$
$$t_h = \frac{\beta}{1+\beta}$$

Since the wife and husband are identical in preferences structure and differ only by wages, the wife's choices are as follows:

$$x_w = \frac{\alpha w}{1+\beta}$$
$$t_w = \frac{\beta}{1+\beta}$$

The indirect utility functions of the wife (V_0^w) and husband (V_0^h) are as follows:

$$\begin{aligned} V_0^h &= \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \\ V_0^w &= \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \end{aligned}$$

For the Nash Bargaining solution we have $R_h = V_0^h$ and $R_w = V_0^w$. The household's optimization problem is hence given as follows:

$$\max_{x_w,x_h,T} N_N = \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1-t_h)w + (1-t_w)\alpha w \\ T &= t_w + t_h \end{aligned}$$

The following proposition characterizes the households decision making problem in the non-patriarchal regime in the presence of conflict of preferences between spouses.

Proposition 2. *When there is conflict of preferences between the wife and husband in the non-patriarchal regime, labour supply is determined by considerations of efficiency. The spouse who earns more will always work full time irrespective of gender. The other spouse will either do full time domestic work, or split her/his time between market and domestic work. The labour supply and utilities of the wife $\{l_w, V_w\}$ and husband $\{l_h, V_h\}$ as a function of the wife's relative wage α is as follows:*

$$l_w = \begin{cases} 0 & if\ \alpha \in (0,\beta] \\ \frac{1-\frac{\beta}{\alpha}}{\beta+1} & if\ \alpha \in (\beta,1) \\ 1 & if\ \alpha \in [1,\infty) \end{cases}$$

$$V_w = \begin{cases} \frac{\left[(1+\beta)^{1+\beta}-(1-\alpha)\beta^\beta\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in (0,\beta] \\ \frac{\left[(1+\alpha)^{1+\beta}-(1-\alpha)\alpha^\beta\right]\beta^\beta w}{2\alpha^\beta(1+\beta)^{1+\beta}} & if\ \alpha \in (\beta,1) \\ \frac{\left[(1+\alpha)^{1+\beta}-1+\alpha\right]\beta^\beta w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1,\frac{1}{\beta}\right) \\ \frac{\left[\alpha(1+\beta)^{1+\beta}-(1-\alpha)\beta^\beta\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta},\infty\right) \end{cases}$$

$$l_h = \begin{cases} 1 & if\ \alpha \in (0,1) \\ \frac{1-\alpha\beta}{1+\beta} & if\ \alpha \in \left[1,\frac{1}{\beta}\right) \\ 0 & if\ \alpha \in \left[\frac{1}{\beta},\infty\right) \end{cases}$$

$$V_h = \begin{cases} \frac{\left[(1+\beta)^{1+\beta}+(1-\alpha)\beta^\beta\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in (0,\beta] \\ \frac{\left[(1+\alpha)^{1+\beta}+(1-\alpha)\alpha^\beta\right]\beta^\beta w}{2\alpha^\beta(1+\beta)^{1+\beta}} & if\ \alpha \in (\beta,1) \\ \frac{\left[(1+\alpha)^{1+\beta}+1-\alpha\right]\beta^\beta w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1,\frac{1}{\beta}\right) \\ \frac{\left[\alpha(1+\beta)^{1+\beta}+(1-\alpha)\beta^\beta\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta},\infty\right) \end{cases}$$

The detailed proof of this proposition is given in Mathematical Appendix A. Each spouse's labour supply under collective decision making is shown in figures 3 and 4.

We note here that the labour supply curves of the spouses, despite the conflict of

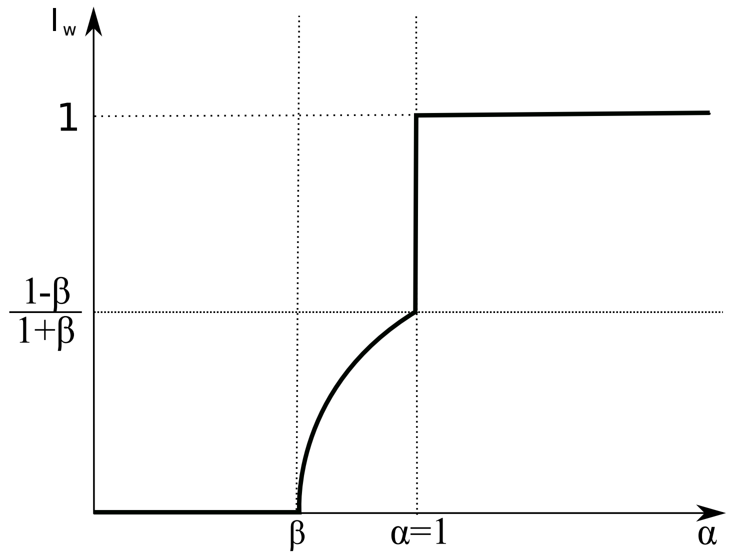

Figure 3: The wife's market labour supply (l_w) as a function of her relative wage α under the non-patriarchal regime.

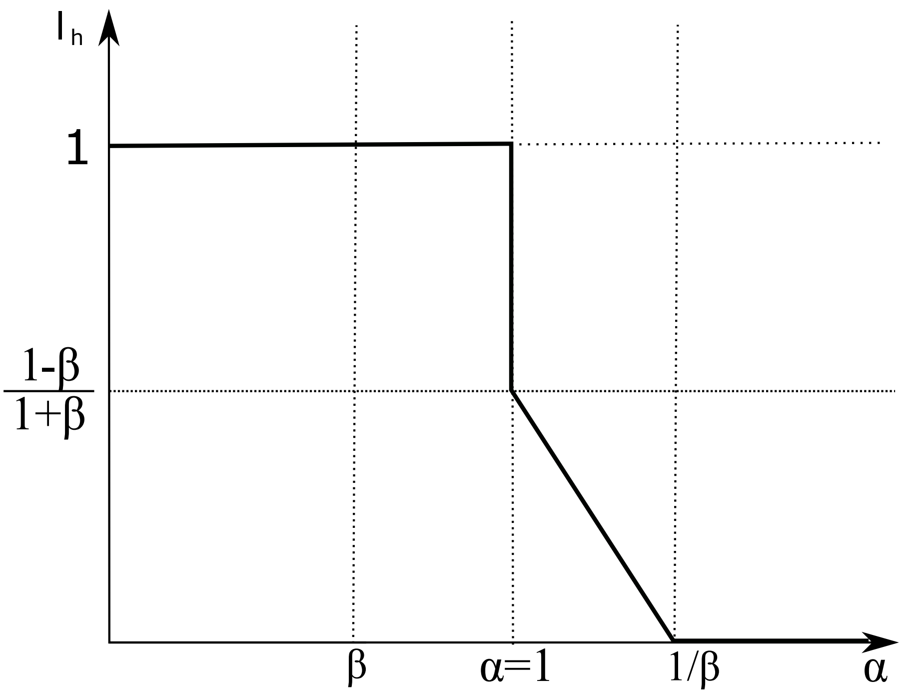

Figure 4: The husband's market labour supply (l_h) as function of the wife's relative wage α under the non-patriarchal regime.

preferences, are identical to when there is no conflict of preference. The reason that the labour supply curves are identical is because both solutions provide the same household welfare maximizing income that places the household on the pareto frontier. Although the labour supply curves are identical, the welfare of the wife and husband will differ in the two cases. Further, the indirect utilities that have been calculated in the solution will help us solve the first stage of the household decision making problem of the patriarchal regime.

4.2 The Patriarchal Regime

In this section, we explore household decision making in societies with a culture of classical patriarchy characterized by the subordination of women. The subordination of women is rooted in the gender based division of work with men engaging in paid work and women being restricted to the household sphere. In this regime, when compared to the non-patriarchal regime, the husband has the option of forcing the wife to stay home full time and not participate in the labour force (Derné, 1994). Whether or not he exercises this option depends on its implications on his welfare. Given this additional control variable which the husband has access to, we pose household decision making as a two stage problem. The first stage involves a choice on the part of the husband to either let the wife work or choose a gender based division of work by preventing her from working, in order to maximize his welfare. The second stage involves choosing time allocated to household work and private consumption contingent on the first stage. If the husband chooses the former option, the household's decision making problem is identical to the non-patriarchal regime. If he chooses the latter option, he engages in the labour force and supplies residual time to household household work while the wife is restricted to doing household work. In this case, the husband's threat point remains the same as in the non-patriarchal regime. However, the wife's threat point shifts to 0. The threat point of the wife shifting to 0 can be theorized in multiple ways. Once married, the husband's choice of preventing the wife from working excludes her from the relevant employment networks which might prevent her from returning to the labour force if she breaks away from marriage. Another factor that can be used to argue for the loss of bargaining power is that, in societies having a social structure of classical patriarchy, the micro-traditions that allow men to prevent the wife from working also stigmatize divorce. Social ire falling disproportionately on women when compared to men which reduces their welfare from the outside option of being divorced to zero. Further, we also analyze the welfare of the wife in this regime, and compare it her welfare in the non-patriarchal regime as well as her welfare when single.

We solve the problem by backward induction. The second stage involves household decision making contingent on whether gender based division of work is chosen in the first stage. If the husband chooses the gender neutral division of work, the household decision making problem is identical to the non-patriarchal solution discussed in the prevision section. On the other hand if the husband chooses the gender based division of work, the household decision making problem is again posed as a bargaining problem and solved using the Nash Bargaining solution. The household decision making involves finding the supply of household work by the husband and division of the household income between the wife's and husband's private consumption. The bargaining problem is as follows:

$$\max_{x_w, x_h, t_h} N_P = \left[x_h \cdot (T)^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot (T)^{\beta} \right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1 - t_h)w \\ T &= t_h + 1 \end{aligned}$$

The solution to the above bargaining problem is characterized by the following proposition.

Proposition 3. *When household decisions are made in the patriarchal regime and the husband restricts his wife to the household sphere, the division of labour is perfectly polarized with the husband engaging in paid work full time and not committing any time to household work for the entire domain of the wife's relative wage (α). The labour supply and welfare of the wife $\{l_w, V_w\}$ and husband $\{l_h, V_h\}$* are as follows:

$$\begin{aligned} l_w &= 0 \\ V_w &= \frac{-\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}} \end{aligned}$$

$$\begin{aligned} l_h &= 1 \\ V_h &= \frac{\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}} \end{aligned}$$

Refer to the Mathematical Appendix A for the detailed proof.

When the husband chooses to prevent his wife from working, it is no longer optimal for him to supply any labour to household work. The wife remains out of the labour force no matter how high her wages are relative to that of her husband. Between this solution and the non-patriarchal solution, we have solved the second stage of the decision making completely. The two solutions tell the husband what his welfare will be when he exercises control over his wife's labour market choices. Hence, he chooses to let his wife work or restrict her to the household sphere to further his welfare. The first stage of the household decision making in a patriarchal regime boils down to choosing between a smaller share of larger household welfare pie in the non-patriarchal solution and a larger share of a smaller household welfare pie when he restricts the wife to the household sphere, depending on which share is larger.

4.2.1 The husband's choice between the gender based and gender neutral solutions.

Here, we solve the first stage of the household decision making problem and compare his welfare in the two solutions for different values of his wife's relative wage α. The final choice of the husband depends on whichever solution gives him a higher welfare. Solving the first stage provides us the first theorem of this study which provides the essence of the effect of the patriarchal regime on the women's labour market outcomes.

Theorem 1. *When household decisions are made in the patriarchal regime, labour supply is determined by considerations of the husband's welfare. The husband faces the dilemma* *of letting the wife work and enjoying a higher household income, or preventing the wife from entering the labour force to gain bargaining power in the household at the cost of a lower household income. As a result, in the patriarchal regime, under certain conditions, women will be working at home full time even though it would be efficient for them to join the labour force.*

Refer to the Mathematical Appendix A for the detailed proof.

Proof. From the welfare levels of the husband calculated in Proposition 2 and 3, we make welfare comparisons for the entire domain of the wife's relative wage (α).

CASE 1: $\alpha \in (0, \beta]$

The husband's indirect utility under the patriarchal solution, and denoted by subscript Ph:

$$V_{Ph} = \frac{[\beta^{\beta} + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

The husband's indirect utility under the non-patriarchal solution, and denoted by subscript Nh:

$$V_{Nh} = \frac{\left[(1+\beta)^{1+\beta} + (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}}$$

Comparing the indirect utilities of the husband, we find:

$$V_{Ph} - V_{Nh} = \frac{\alpha\beta^{\beta} w}{(1+\beta)^{1+\beta}} > 0 \tag{5}$$

We hence see that $V_{Ph} > V_{Nh}$ for all α in this range.

CASE 2: $\alpha \in (\beta, 1)$

The husband's indirect utility under the patriarchal solution, denoted by subscript Ph:

$$V_{Ph} = \frac{[\beta^{\beta} + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

The husband's indirect utility under the non-patriarchal solution, denoted by subscript Nh:

$$V_{Nh} = \frac{\left[(1+\alpha)^{1+\beta} + (1-\alpha)\alpha^{\beta}\right] \beta^{\beta} w}{2\alpha^{\beta}(1+\beta)^{1+\beta}}$$

Comparing the indirect utilities of the husband, we find:

$$V_{Ph} - V_{Nh} = \left[\frac{(1+\beta)^{1+\beta}}{\beta^{\beta}} + \alpha - \frac{(1+\alpha)^{1+\beta}}{\alpha^{\beta}}\right] \frac{\beta^{\beta} w}{2(1+\beta)^{1+\beta}} \tag{6}$$

Using numerical methods, we show that $V_{Ph} > V_{Nh}$ for all α in this range. Refer to Appendix B for the detailed numerical analysis.

CASE 3: $\alpha \in \left[1, \frac{1}{\beta}\right)$

The husband's indirect utility under the patriarchal solution, denoted by subscript Ph:

$$V_{Ph} = \frac{[\beta^\beta + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

The husband's indirect utility under the non-patriarchal solution, denoted by subscript Nh:

$$V_{Nh} = \frac{\left[(1+\alpha)^{1+\beta} + 1 - \alpha\right]\beta^\beta w}{2(1+\beta)^{1+\beta}}$$

Comparing the indirect utilities of the husband, we find:

$$V_{Ph} - V_{Nh} = \left[\frac{(1+\beta)^{1+\beta}}{\beta^\beta} + \alpha - (1+\alpha)^\beta\right]\frac{\beta^\beta w}{2(1+\beta)^{1+\beta}} \tag{7}$$

Using numerical methods, we can show that $V_{Ph} > V_{Nh}$ if and only if $\alpha \leq \overline{\alpha}$ in this range. We see that $\overline{\alpha} > 1$. Further, it is less than $\frac{1}{\beta}$ only for $\beta \in (0, 0.68)$. Refer to Appendix B for the detailed numerical analysis.

CASE 4: $\alpha \in \left[\frac{1}{\beta}, \infty\right)$

The husband's indirect utility under the patriarchal solution, denoted by subscript Ph:

$$V_{Ph} = \frac{[\beta^\beta + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

The husband's indirect utility under the non-patriarchal solution, denoted by subscript Nh:

$$V_{Nh} = \frac{\left[\alpha(1+\beta)^{1+\beta} + (1-\alpha)\beta^\beta\right]w}{2(1+\beta)^{1+\beta}}$$

Comparing the indirect utilities of the husband, we find:

$$V_{Ph} - V_{Nh} = \left\{(1+\beta)^{1+\beta} - \alpha\left[(1+\beta)^{1+\beta} - \beta^\beta\right]\right\}\frac{w}{2(1+\beta)^{1+\beta}} \tag{8}$$

$V_{Ph} > V_{Nh}$ if only if:

$$\alpha \leq \frac{(1+\beta)^{1+\beta}}{(1+\beta)^{1+\beta} - \beta^\beta} \equiv \overline{\alpha}$$

Plotting $\overline{\alpha}$ as a function of β, we find that $\overline{\alpha} \geq \frac{1}{\beta}$ only for $\beta \in (0.68, 1)$. Refer to Appendix B for the plot of $\overline{\alpha}$ vs β.

We now summarize the wife's and husband's labour supply ($l_i = 1 - t_i,\ i = w, h$) under the patriarchal regime. If $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$:

$$l_w = \begin{cases} 0 & if\ \alpha \in (0, \overline{\alpha}] \\ 1 & if\ \alpha \in (\overline{\alpha}, \infty) \end{cases}$$

$$l_h = \begin{cases} 1 & if \ \alpha \in (0, \overline{\alpha}] \\ \frac{1-\alpha\beta}{1+\beta} & if \ \alpha \in \left(\overline{\alpha}, \frac{1}{\beta}\right) \\ 0 & if \ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases}$$

Figures 5 and 6 provide a graphical description of this case.

If $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$:

$$l_w = \begin{cases} 0 & if \ \alpha \in (0, \overline{\alpha}] \\ 1 & if \ \alpha \in (\overline{\alpha}, \infty) \end{cases}$$

$$l_h = \begin{cases} 1 & if \ \alpha \in (0, \overline{\alpha}) \\ 0 & if \ \alpha \in [\overline{\alpha}, \infty) \end{cases}$$

□

Figures 7 and 8 provide a graphical description of this case. We see that for women's relative wage between β and $\overline{\alpha}$, women will supply all their time to household work even though from an overall household welfare perspective, they should either engage in paid work partially or fully. Hence, this theorem argues that it is not surprising to observe low levels of FLFP in societies which are characterized by classical patriarchy and hence is key to understanding the state of low FLFP in India.

We now proceed to the analysis of women's welfare in societies characterized by classical patriarchy.

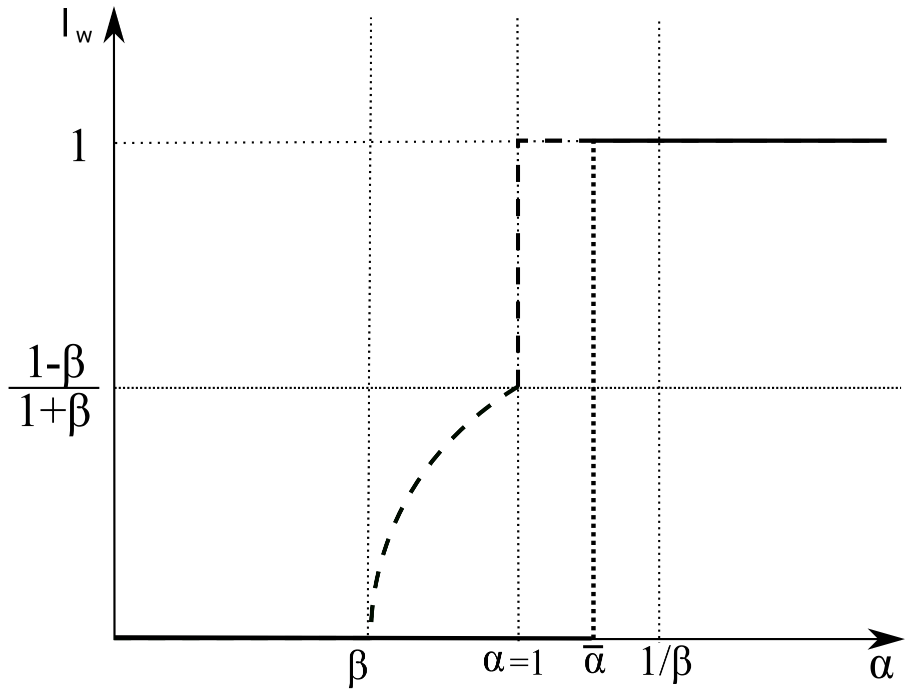

Figure 5: Wife's market labour supply as a function of her relative wage α for $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$. The Dashed line indicates the labour supply in the non-patriarchal regime.

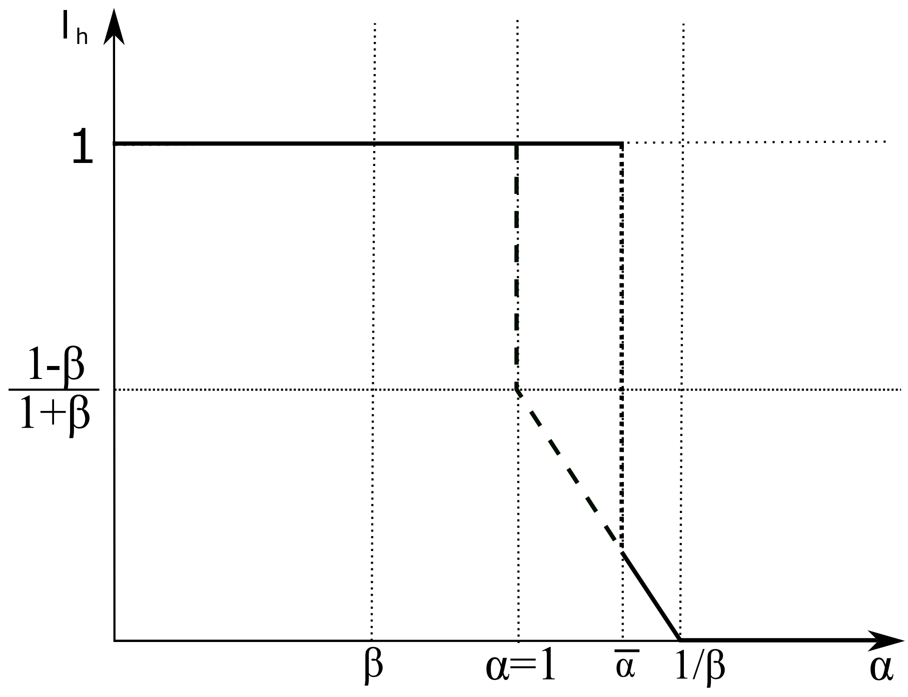

Figure 6: Husband's market labour supply as a function of the wife's relative wage α for $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$. The Dashed line indicates the labour supply in the non-patriarchal regime.

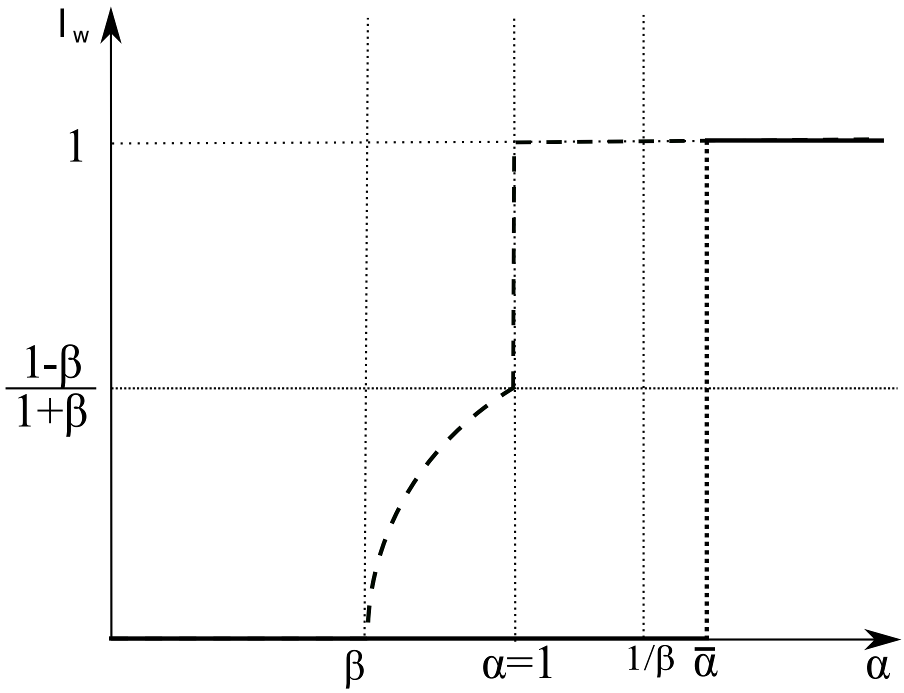

Figure 7: Wife's market labour supply (l_w) as a function of her relative wage α for $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$. The Dashed line indicates the labour supply in the non-patriarchal regime.

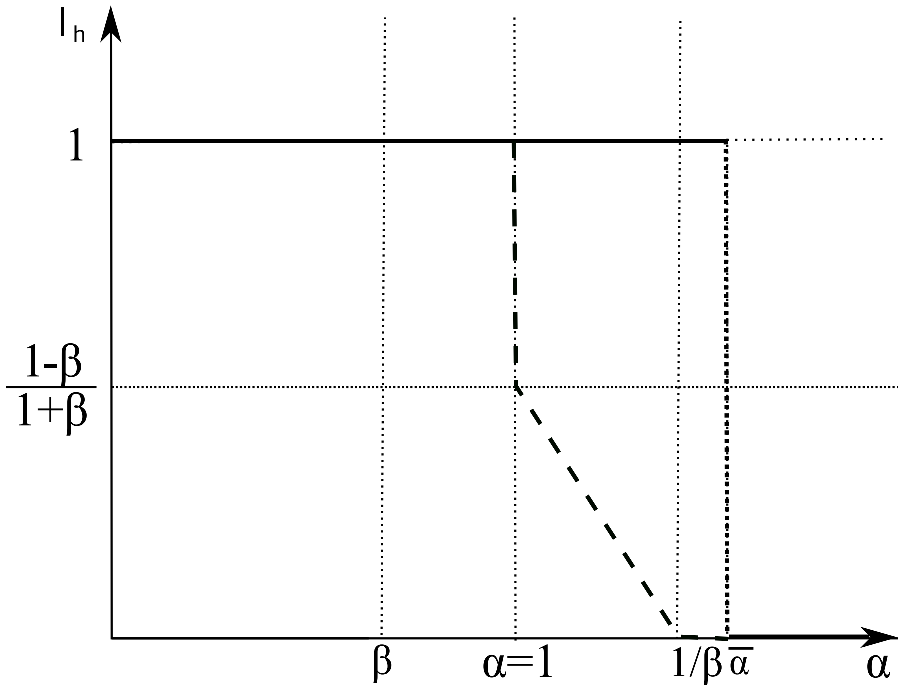

Figure 8: Husband's market labour supply (l_h) as a function of the wife's relative wage α for $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$. The Dashed line indicates the labour supply in the non-patriarchal regime.

4.2.2 Welfare of the wife in the patriarchal regime

The patriarchal solution which maximizes the husband's welfare leaves the wife indifferent when the non-patriarchal solutions is adopted and necessarily worse than in the non-patriarchal system when they are restricted to the household sphere. The reason for women being worse in marriage when they are restricted to the household sphere is analyzed for the two cases, as can been seen in figures 5 and 7. The first is for relative wages $\alpha \leq \beta$, in this case even in the non-patriarchal solution the wife doesn't work, but women in the patriarchal do still worse because of the loss of bargaining power they face when are are prevented from joining the labour market. For relative wages $\alpha \in (\beta, \overline{\alpha}]$, women in the patriarchal solution do household work full time while their counters parts in non-patriarchal societies engage in paid labour either partially or fully depending on their relative wage. The welfare of women in the patriarchal regime is lower than in the non-patriarchal regime since on one hand the welfare of the household in the patriarchal regime will always be lower because it is not being optimized with respect to the wife's labour supply and on the other hand, from this smaller overall household welfare pie, the husbands share is larger than in the non-patriarchal solution. Although it is clear that women in patriarchal societies do worse than their counter parts in non-patriarchal societies, the question remains whether there are any incentives for women to be in marriage at all. Analyzing this question gives us the following proposition,

Proposition 4. *When household decisions are made in the patriarchal regime, the wife faces a loss of bargaining power and welfare relative to the non-patriarchal regime, whenever she is restricted to the household sphere. Furthermore, she will often not even receive the welfare that she enjoyed when she was single,* $\forall\ \alpha \in (\alpha^D, \overline{\alpha}]$. *Over this range of parameters, she would rationally choose to remain single if society allows that option.*

Refer to the Mathematical Appendix A for the detailed proof.

It is worth noting that although women in patriarchal societies do worse than their counterparts in non-patriarchal societies when they are prevented from joining the labour force, they are still better of than being single as long as their relative wage $\alpha \leq \alpha^D$. At the same time, being married is worse than being single as long as relative wage $\alpha \in (\alpha^D, \overline{\alpha}]$. In this range women might choose not to get married, however they often face severe social stigma for being single in patriarchal societies and hence may not choose to exercise this option. At the same time, it is worth considering whether instead of bargaining between the wife and husband happening in the second stage, once the husband has chosen whether to pick a gender based or gender neutral division of work in the first stage, is it possible that bargaining can happen in the first stage? In other words, is it possible for the wife and husband to agree on the gender neutral division of work at the time of marriage by working out a mutually beneficial arrangement that will eliminate the inefficiency due to the gender based division of work. Here, we assume that such agreements for distribution of gains within marriage are not possible due to the couple lacking access to institutions that enforce household contracts(Lundberg and Pollak, 1994).

So far the analysis shows us that societies characterized by classical patriarchy will typically exhibit lower participation of women in the labour force when compared to non-patriarchal societies. Although this helps us understand the low level of FLFP in India, we need to provide more structure to the model to explain the U-Shape of FLFP with respect to women's education (Klasen and Pieters, 2015).

5 Explaining the U-Shaped FLFP vs Women's Education: A Model with Market Purchased Time in Household Decision Making with Conflict of Preferences

We now extend the model by allowing household time (t_b) to be purchased from the market. This market bought household time is assumed to be a perfect substitute for a couple's own time contributions to household work. Whether or not the household purchases time from the market depends on whether purchasing household help is cheaper than the opportunity cost of not working for the couple.

We assume that marriage markets are characterized by exogenous matching based on education and is such that a matched couple has identical education level. Men in the economy earn wages which are proportional to their education level. The wage function at any point of time is given by:

$$w = \pi e$$

Here, π is a positive constant.

Women, however, face discrimination in the job markets and only earn a fraction α^d of the husband's wage even though they have the same education level. However, we assume that there is a household help sector in the economy which allows households to purchase household time at a price $\overline{w}$. This price also acts as a support wage for women engaged in the labour force. Hence, the relative wage of the wife is α^d when both spouses have education levels that earn them incomes higher than the household help sector price. However, as education based wage level of women falls below $\overline{w}$, the wages of women are preserved at $\overline{w}$ by the household help sector and the wage discrimination declines and at very low education levels of the couple, turns in favour of women. Figure 9 shows the relation between the husband's wage and the relative wage α of the wife. We now analyze the household's labour market choices in each of the two household regimes with the objective of mapping the labour supply of women with respect to their education level and husband's wage rate.

5.1 The Non-Patriarchal Regime

We analyze the household's labour market choices when decision making is in the non-patriarchal regime. We have three cases. The first is when the couple's wage rate is greater than the price of purchasing household help. The second is when the husband's wage rate is greater but the wife's education based wage is less than the price of purchasing household help. The third is when both spouses earn less than the price of purchasing household help.

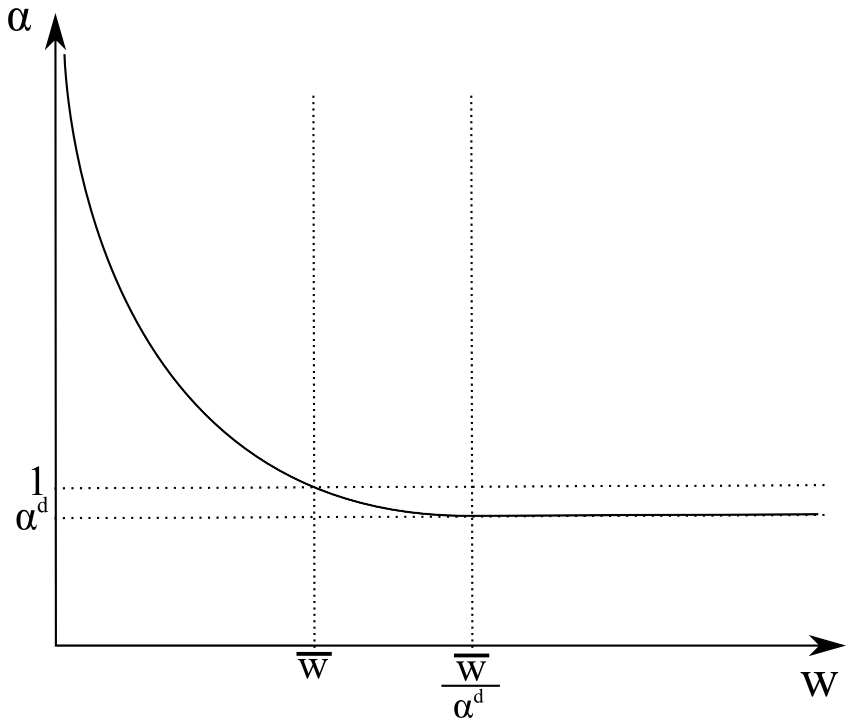

Figure 9: Response of wife's relative wage α to husband's wage w.

5.1.1 The price of purchasing household time is less than the couple's market wages, $\overline{w} < w$ and $\overline{w} < \alpha^d w$.

In this case, the couple will choose to work full time in the market and purchase all the required household time from the market. The household time that is purchased from the market is denoted by t_b. Since the wife and husband earn more than the price of purchasing household time, relative wage of the wife is α^d. Since the household decision making problem is solved using the Nash Bargaining Solution, we first find each spouse's threat-point which is derived from the following optimization exercises:

$$\max_{x_h, t_b} x_h \cdot t_b^{\beta} \quad s.t. \;\; x_h + \overline{w} t_b = w \tag{9}$$

$$\max_{x_w, t_b} x_w \cdot t_b^{\beta} \quad s.t. \;\; x_w + \overline{w} t_b = \alpha^d w \tag{10}$$

Here, the men and women always buy household help from the market than do it themselves if they are not married, since their wages are higher than the price of purchasing household help. Solving this we find the following:
For the man:

$$\begin{aligned} x_h &= \frac{w}{1+\beta} \\ t_b &= \frac{\beta w}{(1+\beta)\overline{w}} \\ V_0^h &= \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \end{aligned}$$

For the woman:

$$x_w = \frac{\alpha^d w}{1+\beta}$$
$$t_b = \frac{\beta\alpha^d w}{(1+\beta)\overline{w}}$$
$$V_0^w = \frac{(\alpha^d)^{1+\beta}\left(\frac{\beta w}{\overline{w}}\right)^{\beta} w}{(1+\beta)^{1+\beta}}$$

The indirect utility of the husband and wife, when single, gives us their threat points. This allows us to define the household's decision making problem which is as follows:

$$\max_{x_w,x_h,t_b} N = \left[x_h \cdot t_b^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot t_b^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta}(\alpha^d)^{1+\beta} w}{(1+\beta)^{1+\beta}}\right]$$

s.t.

$$x_w + x_h + \overline{w}t_b = (1+\alpha^d)w$$

Since both spouses earn more than the cost of hiring household help, they will work full time and purchase all the household help they need from the market. Solving the problem for the optimal purchase of household help (t_b), and the welfare of the wife (V_w) and husband (V_h), we find the following :

$$t_b = \frac{\beta w(1+\alpha^d)}{\overline{w}(1+\beta)}$$
$$V_w = \frac{1}{2}\left[(1+\alpha^d)^{1+\beta} - (1-(\alpha^d)^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$
$$V_h = \frac{1}{2}\left[(1+\alpha^d)^{1+\beta} + (1-(\alpha^d)^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

The detailed proof is provided in the Mathematical Appendix A.

5.1.2 The price of purchasing household time is more than the wife's market wages and less than the husband's wages, $\alpha^d w \leq \overline{w}$ and $\overline{w} < w$.

Since the wife's wage is supported by the household help sector price, the couple is indifferent between purchasing household help or the wife supplying it herself. We assume that she chooses to work full time in the market and purchase all the needed household help from the market. The household decision making problem is identical to the previous case with the only difference that the relative wage $\alpha = \frac{\overline{w}}{w} > \alpha^d$. Hence we have:

$$\begin{aligned}
t_b &= \frac{\beta w(1+\alpha)}{\overline{w}(1+\beta)} \\
x_w &= \frac{1}{2}\left[(1+\alpha)^{1+\beta} - (1-\alpha^{1+\beta})\right]\frac{w}{(1+\alpha)^{\beta}(1+\beta)} \\
x_h &= \frac{1}{2}\left[(1+\alpha)^{1+\beta} + (1-\alpha^{1+\beta})\right]\frac{w}{(1+\alpha)^{\beta}(1+\beta)} \\
V_w &= \frac{1}{2}\left[(1+\alpha)^{1+\beta} - (1-\alpha^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \\
V_h &= \frac{1}{2}\left[(1+\alpha)^{1+\beta} + (1-\alpha^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}
\end{aligned}$$

5.1.3 The price of purchasing household time is more than wage rate of both spouses, $\alpha^d w < \overline{w}$ and $w \leq \overline{w}$.

Here, $\alpha = \frac{\overline{w}}{w} > 1$. As the husband's education and linked wage falls, α keeps rising and the analysis is identical to the decision making problem of the household when $\alpha > 1$ and there is no household help sector in the non-patriarchal regime. The analysis can be found in the proof of *Proposition 2* in the Mathematical Appendix A.

Considering that $w = \frac{\overline{w}}{\alpha}$ we see that:

$$l_h = \begin{cases} \frac{1-\alpha\beta}{1+\beta} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \quad or\ w \in (\beta\overline{w}, \overline{w}] \\ 0 & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \quad or\ w \in (0, \beta\overline{w}] \end{cases}$$

$$l_w = 1 \qquad if\ \alpha \in [1, \infty)\ or\ w \in (0, \overline{w}]$$

$$V_h = \begin{cases} \frac{\left[(1+\alpha)^{1+\beta}+1-\alpha\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \quad or\ w \in (\beta\overline{w}, \overline{w}] \\ \frac{\left[\alpha(1+\beta)^{1+\beta}+(1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \quad or\ w \in (0, \beta\overline{w}] \end{cases}$$

$$V_w = \begin{cases} \frac{\left[(1+\alpha)^{1+\beta}-1+\alpha\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \quad or\ w \in (\beta\overline{w}, \overline{w}] \\ \frac{\left[\alpha(1+\beta)^{1+\beta}-(1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \quad or\ w \in (0, \beta\overline{w}] \end{cases}$$

Summarizing the results of the non-patriarchal regime and replacing $\alpha = \frac{\overline{w}}{w}$ we have the following:

$$l_h = \begin{cases} 0 & if\ w \in (0, \beta\overline{w}] \\ \frac{1-\frac{\overline{w}\beta}{w}}{1+\beta} & if\ w \in (\beta\overline{w}, \overline{w}] \\ 1 & if\ w \in (\overline{w}, \infty) \end{cases}$$

$$l_w = 1 \qquad \forall\ w \in (0, \infty)$$

$$V_h = \begin{cases} \frac{\left[\frac{\overline{w}}{w}(1+\beta)^{1+\beta} + (1-\frac{\overline{w}}{w})\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \ w \in (0, \beta\overline{w}] \\ \frac{\left[(1+\frac{\overline{w}}{w})^{1+\beta} + 1 - \frac{\overline{w}}{w}\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}} & if\ \ w \in (\beta\overline{w}, \overline{w}] \\ \frac{1}{2}\left[(1+\frac{\overline{w}}{w})^{1+\beta} + 1 - \left(\frac{\overline{w}}{w}\right)^{1+\beta}\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ \ w \in \left(\overline{w}, \frac{\overline{w}}{\alpha^d}\right] \\ \frac{1}{2}\left[(1+\alpha^d)^{1+\beta} + 1 - (\alpha^d)^{1+\beta}\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ \ w \in \left(\frac{\overline{w}}{\alpha^d}, \infty\right) \end{cases}$$

$$V_w = \begin{cases} \frac{\left[\frac{\overline{w}}{w}(1+\beta)^{1+\beta} - 1 + \frac{\overline{w}}{w}\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \ w \in (0, \beta\overline{w}] \\ \frac{\left[(1+\frac{\overline{w}}{w})^{1+\beta} - 1 + \frac{\overline{w}}{w}\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}} & if\ \ w \in (\beta\overline{w}, \overline{w}] \\ \frac{1}{2}\left[(1+\frac{\overline{w}}{w})^{1+\beta} - 1 + \left(\frac{\overline{w}}{w}\right)^{1+\beta}\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ \ w \in \left(\overline{w}, \frac{\overline{w}}{\alpha^d}\right] \\ \frac{1}{2}\left[(1+\alpha^d)^{1+\beta} - 1 + (\alpha^d)^{1+\beta}\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ \ w \in \left(\frac{\overline{w}}{\alpha^d}, \infty\right) \end{cases}$$

The key inference from household decision making in this section is that women in non-patriarchal household will always supply all their time to labour markets as long as there is market which allows them purchase household help and also provides them with supporting wages.

5.2 The Patriarchal Regime

In the patriarchal regime, the husband has the option of enforcing a gender based division of work with the wife supplying household help full time while he engages in the labour market. Any residual demand for household time is met by the husband or purchased from the market depending on its affordability. Here again we pose the household decision making problem as a two stage decision making problem where in the first stage the husband chooses between an gender neutral and a gender based division of labour to maximize his welfare, and in the second stage, contingent on the first stage, the household makes decisions on each spouse's/husband's time allocation, private consumption, and household time to be purchased from the market.
We solve the problem by backward induction. In the second stage, the household makes decisions depending on the choice of a gender neutral or a gender based division of work as chosen by the husband in the first stage. The solution to household's problem, when the husband chooses the gender neutral division of work, is identical to the solution to the household decision making problem in the non-patriarchal regime. We now solve for the case when the husband chooses to restrict the wife to the household sphere. We analyze this for two cases, the first when the price of purchasing household time is more than the husband's wages and the second when the price of purchasing household time is less than the husband's wages. We deal with these cases in the following sections.

5.2.1 The price of purchasing household time is more than the husband's wages, $w \leq \overline{w}$

In this range, as the husband's education and corresponding wage keeps falling, α keeps rising. This corresponds to the patriarchal regime in *Theorem 1* when $\alpha > 1$. The couple's labour supply is as follows:

If $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$:

$$l_w = \begin{cases} 1 & if\ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 0 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{w}\right] \end{cases}$$

$$l_h = \begin{cases} 0 & if\ w \in (0, \beta\overline{w}] \\ \frac{1-\frac{\beta\overline{w}}{w}}{1+\beta} & if\ w \in \left(\beta\overline{w}, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 1 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{w}\right] \end{cases}$$

If $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$:

$$l_w = \begin{cases} 1 & if\ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 0 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{w}\right] \end{cases}$$

$$l_h = \begin{cases} 0 & if\ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 1 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{w}\right] \end{cases}$$

5.2.2 The price of purchasing household time is less than the husband's wages, $\overline{w} < w$.

Here, the relative wage of the wife is $\alpha \in [\alpha^d, 1)$. In this case, we assume that the husband restricts the wife to the household sphere and engages in the labour markets. Any additional household time over and above the wife's contribution is purchased from the market. Here, the husband's threat point remains the same as in the corresponding case in the non-patriarchal regime. However, when the husband prevents the wife from entering the labour force, her threat point falls to zero. Hence, the couple's threat points are as follows:

$$\begin{aligned} V_0^h &= \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \\ V_0^w &= 0 \end{aligned}$$

The household decision making problem is hence given as follows:

$$\max_{x_w, x_h, t_b} \left[x_h \cdot (T)^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}\right] \left[x_w \cdot (T)^\beta\right]$$

s.t.

$$\begin{aligned} x_w + x_h + \overline{w}t_b &= w \\ T &= 1 + t_b \end{aligned}$$

The following proposition characterizes the solution to the household decision making problem:

Proposition 5. *In the patriarchal regime, under the conditions that household help can be purchased from the market, the husband's wage is higher than the cost of purchasing household help, and he chooses to restrict the wife to the household sphere, the household will purchase help from the market only if the husband's wage, $w > \frac{\overline{w}}{\beta}$.*
The household help purchased from the market (t_b), the welfare of the wife (V_w) and husband (V_h) as a function of the husband's wage are as follows:

$$t_b = \begin{cases} 0 & if\ w \leq \frac{\overline{w}}{\beta} \\ \frac{\frac{\beta w}{\overline{w}} - 1}{1+\beta} & if\ w > \frac{\overline{w}}{\beta} \end{cases}$$

$$V_w = \begin{cases} \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\left(\frac{\beta w}{\overline{w}}\right)^{\beta}} - 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w \leq \frac{\overline{w}}{\beta} \\ \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} - 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w > \frac{\overline{w}}{\beta} \end{cases}$$

$$V_h = \begin{cases} \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\left(\frac{\beta w}{\overline{w}}\right)^{\beta}} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w \leq \frac{\overline{w}}{\beta} \\ \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w > \frac{\overline{w}}{\beta} \end{cases}$$

The detailed proof is given in the Mathematical Appendix A.

5.2.3 The husband's choice between the gender based and gender neutral solutions when $\overline{w} < w$.

Having solved the second stage of the household decision making problem, we move to solving the husband's problem in the first stage. The husband has the option of preventing his wife's participation in the labour force, but he chooses this option over the non-patriarchal solution only if it increases his welfare. Hence, we compare the husband's welfare under the patriarchal solution with his welfare under the non-patriarchal solution to identify which one he will choose. The woman's utility when she is restricted to the household sphere may or may not exceed her reservation utility when she is single. Despite this, we assume that she is married because of the exogenous nature of match making in the marriage market which doesn't give her an option of being single. To keep the analysis simple, we assume that $\alpha^d < \beta$. The comparison of the husband's welfare, for different ranges of w, in both regimes is as follows:

CASE 1: $w \in \left(\overline{w}, \frac{\overline{w}}{\beta}\right]$

The indirect utility of the husband when he restricts the wife to household sphere, denoted by the subscript Ph is:

$$V_{Ph} = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{(\frac{\beta w}{\overline{w}})^{\beta}} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

The indirect utility to the husband under the non-patriarchal solution, denoted by the subscript Nh is:

$$V_{Nh} = \frac{1}{2}\left[\left(1+\frac{\overline{w}}{w}\right)^{1+\beta} + 1 - \left(\frac{\overline{w}}{w}\right)^{1+\beta}\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

Comparing the two, we find that:

$$V_{Ph} - V_{Nh} = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{(\frac{\beta w}{\overline{w}})^{\beta}} - \left(1+\frac{\overline{w}}{w}\right)^{1+\beta} + \left(\frac{\overline{w}}{w}\right)^{1+\beta}\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

$V_{Ph} - V_{Nh} \geq 0$ iff:

$$\left(1+\frac{1}{\beta}\right)^{1+\beta}\beta\left(\frac{\overline{w}}{w}\right)^{\beta} - \left(1+\frac{\overline{w}}{w}\right)^{1+\beta} + \left(\frac{\overline{w}}{w}\right)^{1+\beta} \geq 0$$

The above relation is true and is proved numerically in the Numerical Appendix B. Hence, in this range preventing the wife from engaging in the labour force dominates the option of allowing the wife to participate in the labour force for the husband.

CASE 2: $w \in \left(\frac{\overline{w}}{\beta}, \frac{\overline{w}}{\alpha^d}\right)$

The indirect utility of the husband when he restricts the wife to the household sphere, denoted by the subscript Ph is:

$$V_{Ph} = \frac{1}{2}\left[\left(1+\frac{\overline{w}}{w}\right)^{1+\beta} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

The indirect utility to the husband under the non-patriarchal solution, denoted by the subscript Nh is:

$$V_{Nh} = \frac{1}{2}\left[\left(1+\frac{\overline{w}}{w}\right)^{1+\beta} + 1 - \left(\frac{\overline{w}}{w}\right)^{1+\beta}\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

Comparing the two, we find that:

$$V_{Ph} - V_{Nh} = \frac{1}{2}\frac{\beta^{\beta}\overline{w}}{(1+\beta)^{1+\beta}} > 0$$

Since $V_{Ph} - V_{Nh} > 0$, the husband will choose to restrict the wife to the household sphere.

CASE 3: $w \in \left[\frac{\overline{w}}{\alpha^d}, \infty\right)$
The indirect utility of the husband when he restricts the wife to the household sphere, denoted by the subscript Ph is:

$$V_{Ph} = \frac{1}{2}\left[\left(1+\frac{\overline{w}}{w}\right)^{1+\beta} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

The indirect utility to the husband under the non-patriarchal solution, denoted by the subscript Nh is:

$$V_{Nh} = \frac{1}{2}\left[\left(1+\alpha^d\right)^{1+\beta} + 1 - \left(\alpha^d\right)^{1+\beta}\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

Comparing the two, we find that:

$$V_{Ph} - V_{Nh} = \frac{1}{2}\left[\left(1+\frac{\overline{w}}{w}\right)^{1+\beta} - \left(1+\alpha^d\right)^{1+\beta} + \left(\alpha^d\right)^{1+\beta}\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

$V_{Ph} - V_{Nh} \geq 0$ iff,

$$w \leq \frac{\overline{w}}{\left[(1+\alpha^d)^{1+\beta} - (\alpha^d)^{1+\beta}\right]^{\frac{1}{1+\beta}} - 1} \equiv \overline{\delta}\overline{w}, \quad \left[\overline{\delta} = \frac{1}{\left[(1+\alpha^d)^{1+\beta} - (\alpha^d)^{1+\beta}\right]^{\frac{1}{1+\beta}} - 1}\right]$$

The above relation tells us that as the education and wage rate of the couple keeps rising, opportunity cost for the husband in restricting the wife to the household sphere keeps rising and beyond a level of the couple's wage $w = \overline{\delta}\overline{w}$, the husband no longer restricts the wife to the household sphere and adopts the non-patriarchal solution.

Summarizing the results of this section, the couple's labour supply in the patriarchal regime where household help can be purchased from the market, is as follows:

If $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$:

$$l_h = \begin{cases} 0 & if\ w \in (0, \beta\overline{w}] \\ \frac{1-\frac{\beta\overline{w}}{w}}{1+\beta} & if\ w \in \left(\beta\overline{w}, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 1 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 1 & if\ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 0 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{\delta}\overline{w}\right] \\ 1 & if\ w \in \left(\overline{\delta}\overline{w}, \infty\right) \end{cases}$$

If $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$:

$$l_h = \begin{cases} 0 & if \ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 1 & if \ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 1 & if \ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 0 & if \ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{\delta}\overline{w}\right] \\ 1 & if \ w \in \left(\overline{\delta}\overline{w}, \infty\right) \end{cases}$$

Since the wage rate is a function of education i.e. $w = \pi e$, we can map the couple's labour supply with respect to their common education level as well. This is as follows if $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$:

$$l_h = \begin{cases} 0 & if \ e \in (0, \frac{\beta\overline{w}}{\pi}] \\ \frac{1-\frac{\beta\overline{w}}{w}}{1+\beta} & if \ e \in \left(\frac{\beta\overline{w}}{\pi}, \frac{\overline{w}}{\pi\overline{\alpha}}\right) \\ 1 & if \ e \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 1 & if \ e \in \left(0, \frac{\overline{w}}{\pi\overline{\alpha}}\right) \\ 0 & if \ e \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \frac{\overline{\delta}\overline{w}}{\pi}\right] \\ 1 & if \ e \in \left(\frac{\overline{\delta}\overline{w}}{\pi}, \infty\right) \end{cases}$$

If $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$:

$$l_h = \begin{cases} 0 & if \ w \in \left(0, \frac{\overline{w}}{\pi\overline{\alpha}}\right) \\ 1 & if \ w \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 1 & if \ e \in \left(0, \frac{\overline{w}}{\pi\overline{\alpha}}\right) \\ 0 & if \ e \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \frac{\overline{\delta}\overline{w}}{\pi}\right] \\ 1 & if \ e \in \left(\frac{\overline{\delta}\overline{w}}{\pi}, \infty\right) \end{cases}$$

We now focus on the labour supply of the wife as a function of the husband's wage rate and her education level. Figures 10 and 11 plot the wife's labour supply as a function of husband's wage and her education level respectively. Here, we observe that the labour force participation of married women dips to zero for a range of their husband's wages and their own education levels. This brings us to the second theorem of this paper:

Theorem 2. *In societies characterized by patriarchy and where couples can purchase household help from the market, the labour force participation of married women follows a U-shaped pattern with respect to their education and their husband's wage. Married women's participation in the labour force dips to zero when, their own education level,* $e \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \frac{\overline{\delta}\overline{w}}{\pi}\right]$ *and husband's wage,* $w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{\delta}\overline{w}\right]$.

The above theorem thus provides theoretical underpinning to the empirically observed U-shaped female labour force participation of women with respect to their education level. Hence, in patriarchal societies, higher education level of women may not always correspond to higher participation of women in the labour force.

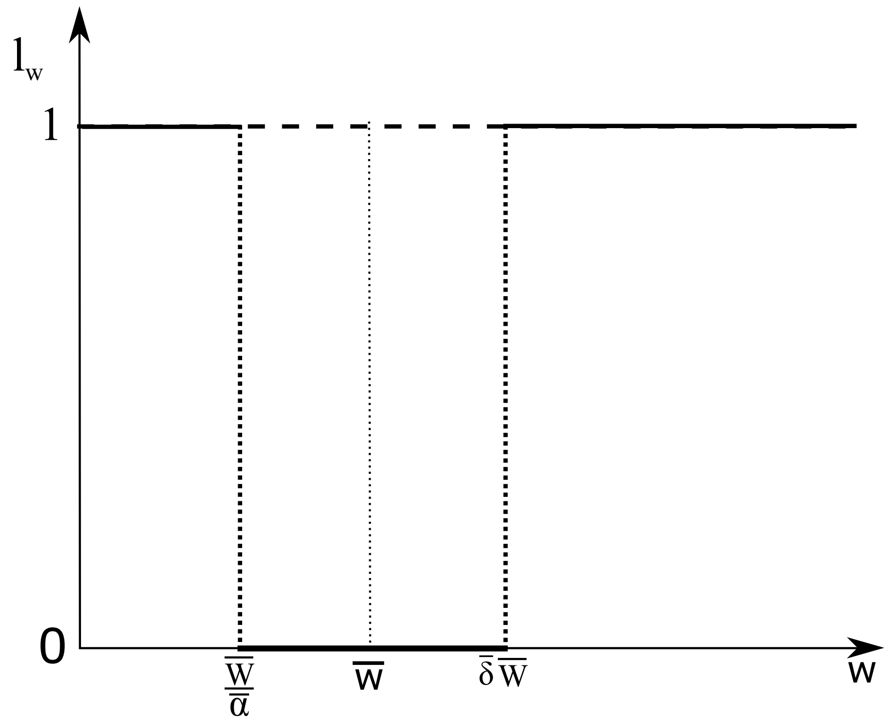

Figure 10: Wife's labour supply (l_w) as a function of her husband's wage rate w in the patriarchal regime. The Dashed line indicates the labour supply in the non-patriarchal regime.

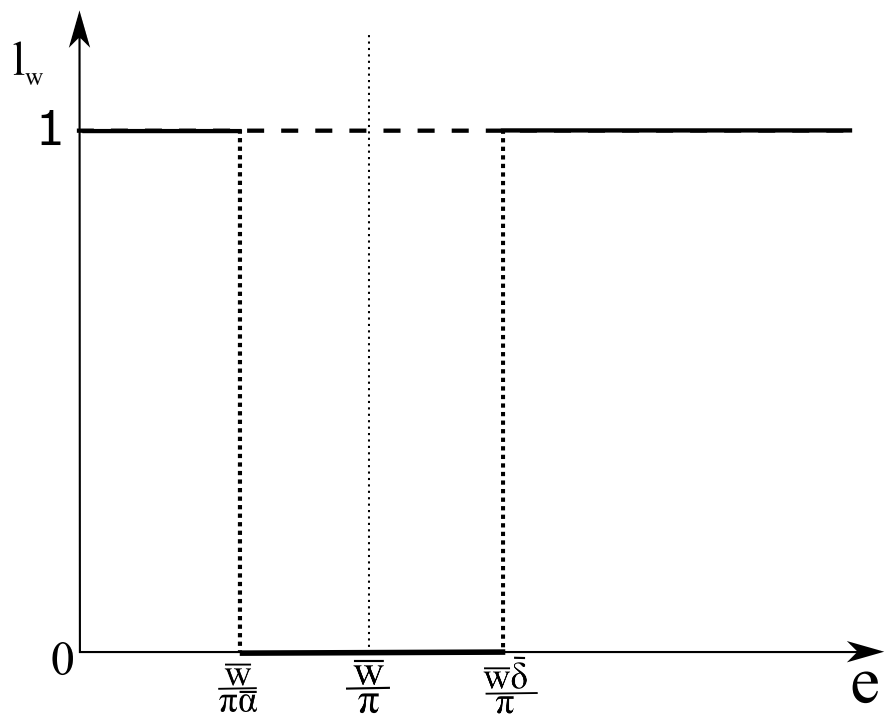

Figure 11: Wife's labour supply (l_w) as a function of her education level e in the patriarchal regime. The Dashed line indicates the labour supply in the non-patriarchal regime.

6 Conclusions

We have two key findings in this chapter. The first is that patriarchal households, where men have the choice between a gender neutral and a gender specific division of work, will choose women's participation in the labour force to maximize the welfare of men and this generates a dead-weight loss in the households. The labour supply that results from this household structure may lead to women not participating in the labour force even if they have a higher earning potential than their husbands. The second is that when we introduce a market for household help, we see that women's labour supply, as a function of her own education and husband's wage, follows a U-shaped, which is observed in the empirical literature. This paper hence provides a theoretical framework that helps explain the puzzling empirical findings on female labour force participation.

Our study suggests that the patriarchal structure of households can have extremely adverse welfare implications for married women and leads to loss of efficiency in economy due to dead-weight losses in households. Thus it is imperative to focus on policies that raise women's bargaining power within the family. Policies that put money in the hands of women will improve efficiency in household decision making. This is because money in the hands of women will help increase their bargaining power in the household, which in turn will reduce the gain to the husband from preventing the wife from working. Hence, this study lends further support to the argument that programs which have conditional cash transfers as incentives should be designed to transfer the money to women. This has the dual benefit of providing incentives to adopt the program as well as reducing dead-weight losses by raising women's say in the household. Further, the U-shaped labour supply curve of women with respect to education tells us that higher education of women need not always lead to higher labour force engagement in patriarchal societies. Unless the education level of women is high enough that it empowers them to break out of the patriarchal bondage to the household sphere, education will not have an unambiguous positive effect on their labour force participation and welfare.

References

Afridi, F., Dinkelman, T., and Mahajan, K. (2017). Why are fewer married women joining the work force in rural india? a decomposition analysis over two decades. *Journal of Population Economics*, pages 1–36.

Afridi, F., Mukhopadhyay, A., and Sahoo, S. (2012). Female labour force participation and child education in india: the effect of the national rural employment guarantee scheme.

Akerlof, G. A. and Kranton, R. E. (2000). Economics and identity. *The Quarterly Journal of Economics*, 115(3):715–753.

Anderson, S. and Eswaran, M. (2009). What determines female autonomy? evidence from bangladesh. *Journal of Development Economics*, 90(2):179–191.

Andres, L. A., Dasgupta, B., Joseph, G., Abraham, V., and Correia, M. (2017). *Precarious drop: Reassessing patterns of female labor force participation in India*. The World Bank.

Basu, K. (2006). Gender and say: A model of household behaviour with endogenously determined balance of power. *The Economic Journal*, 116(511):558–580.

Derné, S. (1994). Hindu men talk about controlling women: Cultural ideas as a tool of the powerful. *Sociological Perspectives*, 37(2):203–227.

Esteve-Volart, B. (2004). Gender discrimination and growth: Theory and evidence from india. *Vol*.

Fletcher, E., Pande, R., and Moore, C. M. T. (2017). Women and work in india: Descriptive evidence and a review of potential policies.

Ghai, S. (2018). The anomaly of women's work and education in india. *Indian Council for Research on International Economic Relations*.

Goldin, C. (1994). The u-shaped female labor force function in economic development and economic history. Technical report, National Bureau of Economic Research.

Kannan, K. and Raveendran, G. (2012). Counting and profiling the missing labour force. *Economic and Political Weekly*, pages 77–80.

Kapsos, S., Bourmpoula, E., Silberman, A., et al. (2014). Why is female labour force participation declining so sharply in india? Technical report, International Labour Organization.

Klasen, S. and Pieters, J. (2015). *What explains the stagnation of female labor force participation in urban India?* The World Bank.

Luke, N. and Munshi, K. (2011). Women as agents of change: Female income and mobility in india. *Journal of Development Economics*, 94(1):1–17.

Lundberg, S. and Pollak, R. A. (1994). Noncooperative bargaining models of marriage. *The American Economic Review*, 84(2):132–137.

McElroy, M. B. and Horney, M. J. (1981). Nash-bargained household decisions: Toward a generalization of the theory of demand. *International economic review*, pages 333–349.

Naidu, S. C. (2016). Domestic labour and female labour force participation. *Economic and Political Weekly*, Vol. 51(Issue No. 44-45).

Osborne, M. J. and Rubinstein, A. (1990). *Bargaining and markets*. Academic press.

World Bank (2019). Global economic prospects: Darkening skies. *Washington, D.C. : World Bank Group*.

Appendices

A Mathematical Appendix

A.1 Proposition 1

When there is no conflict of preferences between the wife and husband, labour supply is determined by considerations of efficiency. The spouse who earns more will always work full time irrespective of gender. The other spouse will either do full time domestic work, or split her/his time between market and domestic work. The labour supply of the wife (l_w) and husband (l_h) as a function of their relative wage α is as follows:

$$l_w = \begin{cases} 0 & if\ \alpha \in (0, \beta] \\ \frac{1-\frac{\beta}{\alpha}}{\beta+1} & if\ \alpha \in (\beta, 1) \\ 1 & if\ \alpha \in [1, \infty) \end{cases}$$

$$l_h = \begin{cases} 1 & if\ \alpha \in (0, 1) \\ \frac{1-\alpha\beta}{1+\beta} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \\ 0 & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases}$$

Proof. The household's optimization problem is given by:

$$\max_{x_w, x_h, t_w, t_h} (x_h)^{\frac{1}{2}} (x_w)^{\frac{1}{2}} T^{\beta}$$

subject to :

$$x_w + x_h = (1 - t_w)\alpha w + (1 - t_h)w$$
$$T = t_w + t_h$$

We set up the Lagrangian for the household's optimization problem which is as follows:

$$\max_{x_w, x_h, t_w, t_h} L = (x_h)^{\frac{1}{2}} (x_w)^{\frac{1}{2}} T^{\beta} + \lambda((1 - t_w)\alpha w + (1 - t_h)w - x_h - x_w)$$

The first derivatives of the Lagrangian are as follows:

$$\frac{\partial L}{\partial x_w} = \frac{1}{2}(x_h)^{\frac{1}{2}}(x_w)^{-\frac{1}{2}} T^{\beta} - \lambda \quad (11)$$
$$\frac{\partial L}{\partial x_h} = \frac{1}{2}(x_h)^{\frac{-1}{2}}(x_w)^{\frac{1}{2}} T^{\beta} - \lambda \quad (12)$$
$$\frac{\partial L}{\partial t_w} = \beta(x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}} T^{\beta-1} - \lambda\alpha w \quad (13)$$
$$\frac{\partial L}{\partial t_h} = \beta(x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}} T^{\beta-1} - \lambda w \quad (14)$$
$$\frac{\partial L}{\partial \lambda} = (1 - t_h)w + (1 - t_w)\alpha w - x_w - x_h \quad (15)$$

Setting the first derivatives to zero, we see that the allocation of time depends on who has the lower cost of household work. Hence, we analyze the following two cases:

1. The wife's wage is less than that of the husband or $\alpha < 1$.

2. The wife's wage is greater than that of the husband or $\alpha \geq 1$.

CASE 1: The wife's wage is less than that of the husband or $\alpha < 1$

In this case the wife makes the first contributions to household work and the husband contributes any residual household time. We start by assuming that the husband works in the market full time and only the wife contributes towards household work (or $t_h = 0$ and $T = t_w$). The constrained optimization problem is solved by setting equations 11, 12, 13 and 15 to zero and we find the following:

$$x_w = x_h = \frac{\alpha w t_w}{2\beta} \tag{16}$$

using this in the budget constraint we find that:

$$t_w = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$$

We know that the wife's time allocation cannot exceed 1 and hence we see that:

$$t_w = \begin{cases} 1 & if\ \alpha \leq \beta \\ \frac{(1+\alpha)\beta}{\alpha(1+\beta)} & if\ \alpha > \beta \end{cases} \tag{17}$$

We now derive the wife's and husband's private consumption for the interior and boundary solution of t_w.

CASE 1.1: The wife's household time allocation has an interior solution, $\alpha > \beta$

In this case we have, $t_h = 0$ and $t_w = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$. Hence, $T = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$. The private consumption levels are given by:

$$x_w = x_h = \frac{(1+\alpha)w}{2(1+\beta)}$$

CASE 1.2: The wife's household time allocation has a boundary solution, $\alpha \leq \beta$

Since $t_w = 1$, we cannot set $\frac{\partial L}{\partial t_w}$ to zero. Hence we set $\frac{\partial L}{\partial t_h}$ to zero and check the conditions under which the husband will start contributing to household work. Hence, now $T = 1 + t_h$, or the total time allocated to household production is full time allocation of the wife and time contributions of the husband. Now, we go back to the first order conditions and set 11, 12, 14 and 15 to zero and we find the following:

$$x_w = x_h = \frac{(1+t_h)w}{2\beta} \tag{18}$$

using this in the budget constraint we find that:

$$t_h = \frac{\beta - 1}{\beta + 1}$$

Since, $\beta < 1$, we are again at a boundary solution of $t_h = 0$ and $T = 1$ for husband's time. We cannot set $\frac{\partial L}{\partial t_h}$ to zero since at this point $\frac{\partial L}{\partial t_h} < 0$. Hence setting 11, 12, and 15 to zero we find:

$$x_w = x_h = \frac{w}{2}$$

CASE 2: The wife's wage is greater than that of the husband or $\alpha \geq 1$

In this case the husband makes the first contributions to household work and any residual demand is contributed by the wife.We start by assuming that the wife works in the market full time and only the husband contributes towards household work (or $t_w = 0$ and $t = t_h$). The constrained optimization problem is solved by setting equations 11, 12, 14 and 15 to zero and we find the following:

$$x_w = x_h = \frac{wt_h}{2\beta} \tag{19}$$

using this in the budget constraint we find that:

$$t_h = \frac{(1+\alpha)\beta}{(1+\beta)}$$

However, we know that the husband's time endowment is 1 and hence:

$$t_h = \begin{cases} 1 & if\ \alpha \geq \frac{1}{\beta} \\ \frac{(1+\alpha)\beta}{(1+\beta)} & if\ \alpha < \frac{1}{\beta} \end{cases} \tag{20}$$

We now derive the wife's and husband's private consumption for the interior and boundary solution of t_h.

CASE 2.1: The husband's household time allocation has an interior solution, $\alpha < \frac{1}{\beta}$

In this case we have, $t_h = \frac{(1+\alpha)\beta}{(1+\beta)}$ and $t_w = 0$. Hence, $T = \frac{(1+\alpha)\beta}{(1+\beta)}$. The private consumption levels are given by:

$$x_w = x_h = \frac{(1+\alpha)w}{2(1+\beta)}$$

CASE 2.2: The husband's household time allocation has a boundary solution, $\alpha \geq \frac{1}{\beta}$

Since $t_h = 1$ we cannot set $\frac{\partial L}{\partial t_h}$ to zero. We set $\frac{\partial L}{\partial t_w}$ to zero to check the conditions under which the wife will start contributing to household work. Hence, $T = 1 + t_w$, or the total time allocated to household production is full time work of the husband and time contributions of the wife. Now, we go back to the first order conditions and set equations 11, 12, 13 and 15 to zero and we find the following:

$$x_w = x_h = \frac{(1+t_w)\alpha w}{2\beta} \tag{21}$$

$$t_w = \frac{\beta - 1}{\beta + 1}$$

However, since $\beta < 1$, t_w cannot fall below 0. Hence, $t_w = 0$ and $T = 1$. Here we again have a boundary solution with respect to t_w and cannot set $\frac{\partial L}{\partial t_w}$ to zero. Setting equations 11, 12 and 15 to zero we find the following:

$$x_w = x_h = \frac{\alpha w}{2}$$

We now summarize the wife's and husband's market work decision, $l_i = 1 - t_i$, *where* $i = w, h$, corresponding to different levels of the wife's relative wage α.

$$l_w = \begin{cases} 0 & if\ \alpha \in (0, \beta] \\ \frac{1-\frac{\beta}{\alpha}}{\beta+1} & if\ \alpha \in (\beta, 1) \\ 1 & if\ \alpha \in [1, \infty) \end{cases} \tag{22}$$

$$l_h = \begin{cases} 1 & if\ \alpha \in (0, 1) \\ \frac{1-\alpha\beta}{1+\beta} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \\ 0 & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases} \tag{23}$$

□

A.2 Proposition 2

When there is conflict of preferences between the wife and husband in the non-patriarchal regime, labour supply is determined by considerations of efficiency. The spouse who earns more will always work full time irrespective of gender. The other spouse will either do full time domestic work, or split her/his time between market and domestic work. The labour supply of the wife (l_w) and husband (l_h) as a function of their relative wage α is as follows:

$$l_w = \begin{cases} 0 & if\ \alpha \in (0, \beta] \\ \frac{1-\frac{\beta}{\alpha}}{\beta+1} & if\ \alpha \in (\beta, 1) \\ 1 & if\ \alpha \in [1, \infty) \end{cases}$$

$$l_h = \begin{cases} 1 & if\ \alpha \in (0, 1) \\ \frac{1-\alpha\beta}{1+\beta} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \\ 0 & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases}$$

Proof. Setting up the Lagrangian we have:

$$\max_{x_w, x_h, t_h, t_w} L_N = \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] + \lambda_\alpha \left[(1-t_h)w + (1-t_w)\alpha w - x_h - x_w\right]$$

The first derivatives of the Lagrangian are as follows:

$$\frac{\partial L_N}{\partial x_w} = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{24}$$

$$\frac{\partial L_N}{\partial x_h} = T^\beta \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{25}$$

$$\frac{\partial L_N}{\partial t_h} = \beta x_w T^{\beta-1} \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h T^{\beta-1} \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda w \tag{26}$$

$$\frac{\partial L_N}{\partial t_w} = \beta x_w T^{\beta-1} \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h T^{\beta-1} \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda \alpha w \tag{27}$$

$$\frac{\partial L_N}{\partial \lambda} = (1-t_h)w + (1-t_w)\alpha w - x_w - x_h \tag{28}$$

The value of α determines who has a lower opportunity cost of supplying household time and hence, is a critical to decision making.

A.2.1 The wife's wage is less than that of the husband or $\alpha < 1$

Revisiting the first derivatives, if $\alpha < 1$, then the wife's opportunity cost of household work is lower and hence, she will take the lead in allocating time towards household work. The Husband will only supply any residual household time if the wife exhausts her time endowment. We start by assuming that $T = t_w$ and $t_h = 0$ and the first derivatives of the Lagrangian are now as follows:

$$\frac{\partial L_N}{\partial x_w} = t_w^\beta \left[x_h \cdot t_w^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{29}$$

$$\frac{\partial L_N}{\partial x_h} = t_w^\beta \left[x_w \cdot t_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{30}$$

$$\frac{\partial L_N}{\partial t_h} = \beta x_w t_w^{\beta-1} \left[x_h \cdot t_w^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h t_w^{\beta-1} \left[x_w \cdot t_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda w \tag{31}$$

$$\frac{\partial L_N}{\partial t_w} = \beta x_w t_w^{\beta-1} \left[x_h \cdot t_w^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h t_w^{\beta-1} \left[x_w \cdot y_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda \alpha w \tag{32}$$

$$\frac{\partial L_N}{\partial \lambda} = w + (1-t_w)\alpha w - x_w - x_h \tag{33}$$

We now set the derivatives of the Lagrangian with respect to x_h, x_w, t_w and λ to zero. Equating equations 29 and 30-

$$t_w^\beta \left[x_w \cdot t_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] = t_w^\beta \left[x_h \cdot t_w^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] \tag{34}$$

or

$$t_w^\beta (x_h - x_w) = (1-\alpha) \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \tag{35}$$

Setting equations 29, 30 and 32 to zero and solving simultaneously we get:

$$x_h + x_w = \frac{\alpha w t_w}{\beta} \tag{36}$$

Using this in the budget constraint we find that:

$$t_w = \frac{(1+\alpha)\beta}{\alpha(1+\beta)} \tag{37}$$

However, the wife's time endowment is capped at 1. Hence we have:

$$t_w = \begin{cases} 1 & if\ \alpha \leq \beta \\ \frac{(1+\alpha)\beta}{\alpha(1+\beta)} & if\ \alpha > \beta \end{cases} \tag{38}$$

We solve for the couple's choice of private goods and their indirect utilities for both the interior and boundary solution.

CASE 1: The wife's household time allocation has an interior solution, $\alpha > \beta$

Here we have $t_h = 0$, $t_w = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$ and $T = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$. Using these in equations 35 and 36 we get:

$$x_h + x_w = \frac{(1+\alpha)w}{1+\beta} \tag{39}$$

$$x_h - x_w = \frac{(1-\alpha)\alpha^\beta w}{(1+\beta)(1+\alpha)^\beta} \tag{40}$$

Solving the above two equations simultaneously we get:

$$x_h = \frac{\left[(1+\alpha)^{1+\beta} + (1-\alpha)\alpha^\beta\right] w}{2(1+\beta)(1+\alpha)^\beta}$$

$$x_w = \frac{\left[(1+\alpha)^{1+\beta} - (1-\alpha)\alpha^\beta\right] w}{2(1+\beta)(1+\alpha)^\beta}$$

The indirect utility functions are hence:

$$V_h = \frac{\left[(1+\alpha)^{1+\beta} + (1-\alpha)\alpha^\beta\right] \beta^\beta w}{2\alpha^\beta(1+\beta)^{1+\beta}}$$

$$V_w = \frac{\left[(1+\alpha)^{1+\beta} - (1-\alpha)\alpha^\beta\right] \beta^\beta w}{2\alpha^\beta(1+\beta)^{1+\beta}}$$

CASE 2: The wife's household time allocation has a boundary solution, $\alpha \leq \beta$

In this case $t_w = 1$ and given that it is a boundary condition we cannot set $\frac{\partial L_N}{\partial t_w}$. We now set $\frac{\partial L_N}{\partial t_h}$ to zero so that we can check if the husband contributes to household work. We proceed by assuming that the wife works at home full time and the residual time demand is supplied by the husband. Now, $T = 1 + t_h$, The first order conditions that are to be solved simultaneously are:

$$\frac{\partial L_N}{\partial x_w} = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{41}$$

$$\frac{\partial L_N}{\partial x_h} = T^\beta \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{42}$$

$$\frac{\partial L_N}{\partial t_h} = \beta x_w T^{\beta-1} \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h T^{\beta-1} \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda w \tag{43}$$

$$\frac{\partial L_N}{\partial \lambda} = (1-t_h)w - x_w - x_h \tag{44}$$

Setting the first order conditions to zero and equating 41 and 42-

$$T^\beta \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] \tag{45}$$

or

$$T^\beta (x_h - x_w) = (1-\alpha) \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \tag{46}$$

Equating equations 41, 42 and 43 to zero and solving simultaneously we get:

$$x_h + x_w = \frac{Tw}{\beta} \tag{47}$$

Using this in the budget constraint we find that:

$$t_h = \frac{\beta - 1}{\beta + 1}$$

However, since $\beta < 1$, we have $t_h = 0$ and $T = 1$. We again have a boundary solution with respect to husband's household time allocation and hence cannot set $\frac{\partial L_N}{\partial t_h}$ to zero since it can be verified to be negative. Setting the first derivatives of the Lagrangian with respect to x_h, x_w and λ to zero we get:

$$x_h + x_w = w \tag{48}$$

Using $T = 1$ in 46 and solving simultaneously with 48 we have:

$$x_h = \frac{\left[(1+\beta)^{1+\beta} + (1-\alpha)\beta^\beta \right] w}{2(1+\beta)^{1+\beta}}$$

$$x_w = \frac{\left[(1+\beta)^{1+\beta} - (1-\alpha)\beta^\beta \right] w}{2(1+\beta)^{1+\beta}}$$

The indirect utility functions are hence:

$$V_h = x_h = \frac{\left[(1+\beta)^{1+\beta} + (1-\alpha)\beta^\beta \right] w}{2(1+\beta)^{1+\beta}}$$

$$V_w = x_w = \frac{\left[(1+\beta)^{1+\beta} - (1-\alpha)\beta^\beta \right] w}{2(1+\beta)^{1+\beta}}$$

A.2.2 The wife's wage is greater than that of the husband or $\alpha \geq 1$

In this case, the wife's opportunity cost of household work is higher than that of the husband. Hence, time allocations to household work are made by the husband first and any residual requirement is supplied by the wife. We start by assuming that $T = t_h$ and $t_w = 0$ and the first derivatives of the Lagrangian are now as follows:

$$\frac{\partial L_N}{\partial x_w} = t_h^\beta \left[x_h \cdot t_h^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{49}$$

$$\frac{\partial L_N}{\partial x_h} = t_h^\beta \left[x_w \cdot t_h^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{50}$$

$$\frac{\partial L_N}{\partial t_h} = \beta x_w t_h^{\beta-1} \left[x_h \cdot t_h^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h t_h^{\beta-1} \left[x_w \cdot t_h^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda w \tag{51}$$

$$\frac{\partial L_N}{\partial t_w} = \beta x_w t_h^{\beta-1} \left[x_h \cdot t_h^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h t_h^{\beta-1} \left[x_w \cdot y_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda \alpha w \tag{52}$$

$$\frac{\partial L_N}{\partial \lambda} = w + (1 - t_h)\alpha w - x_w - x_h \tag{53}$$

Setting the first derivatives with respect to x_h, x_w, t_h and λ to zero and equating 49 and 50 we have :

$$t_h^\beta \left[x_w \cdot t_h^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] = t_h^\beta \left[x_h \cdot t_h^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] \tag{54}$$

or

$$t_h^\beta (x_h - x_w) = (1-\alpha) \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \tag{55}$$

Setting equations 49, 50 and 51 to zero and solving simultaneously we have:

$$x_h + x_w = \frac{t_h w}{\beta} \tag{56}$$

Using this in the budget constraint we find that:

$$t_h = \frac{(1+\alpha)\beta}{1+\beta} \tag{57}$$

However, we know that the husband's time allocation for household time cannot exceed his time endowment of 1 unit. Hence:

$$t_h = \begin{cases} \frac{(1+\alpha)\beta}{1+\beta} & if \ \alpha < \frac{1}{\beta} \\ 1 & if \ \alpha \geq \frac{1}{\beta} \end{cases} \tag{58}$$

We solve for the couple's choice of private goods and their indirect utilities for both the interior and boundary solution.

CASE 1: The husband's household time allocation has an interior solution, $\alpha < \frac{1}{\beta}$

Here, we have $t_w = 0$, $t_h = \frac{(1+\alpha)\beta}{1+\beta}$ and $T = \frac{(1+\alpha)\beta}{1+\beta}$. Replacing these in equations 55 and 56 , we get:

$$\begin{aligned} x_h - x_w &= \frac{(1-\alpha)w}{(1+\beta)(1+\alpha)^{\beta}} \\ x_h + x_w &= \frac{(1+\alpha)w}{1+\beta} \end{aligned}$$

Solving simultaneously for x_h ans x_w we get :

$$x_h = \frac{\left[(1+\alpha)^{1+\beta} + 1 - \alpha\right] w}{2(1+\beta)(1+\alpha)^{\beta}}$$
$$x_w = \frac{\left[(1+\alpha)^{1+\beta} - 1 + \alpha\right] w}{2(1+\beta)(1+\alpha)^{\beta}}$$

The indirect utility functions are hence:

$$V_h = \frac{\left[(1+\alpha)^{1+\beta} + 1 - \alpha\right] \beta^{\beta} w}{2(1+\beta)^{1+\beta}}$$
$$V_w = \frac{\left[(1+\alpha)^{1+\beta} - 1 + \alpha\right] \beta^{\beta} w}{2(1+\beta)^{1+\beta}}$$

CASE 1: The husband's household time allocation has a boundary solution, $\alpha \geq \frac{1}{\beta}$

In this case, we have $t_h = 1$. Given that t_h is at the boundary, we cannot set $\frac{\partial L_N}{\partial t_h}$ to zero. However, we need to check if the wife too contributes to household work. For this we repeat the optimization exercise setting $t_h = 1$ and $T = 1 + t_w$:

$$\max_{x_w, x_h, t_w} L_N = \left[x_h \cdot T^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot T^{\beta} - \frac{\beta^{\beta} \alpha w}{(1+\beta)^{1+\beta}}\right] + \lambda\left[(1-t_w)\alpha w - x_w - x_h\right]$$

FOCs:

$$\frac{\partial L_N}{\partial x_w} = T^{\beta}\left[x_h \cdot T^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{59}$$

$$\frac{\partial L_N}{\partial x_h} = T^{\beta}\left[x_w \cdot T^{\beta} - \frac{\beta^{\beta} \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{60}$$

$$\frac{\partial L_N}{\partial t_w} = \beta x_w T^{\beta-1}\left[x_h \cdot T^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right] + \beta x_h T^{\beta-1}\left[x_w \cdot T^{\beta} - \frac{\beta^{\beta} \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \alpha w \tag{61}$$

$$\frac{\partial L_N}{\partial \lambda} = (1-t_w)\alpha w - x_w - x_h \tag{62}$$

Setting the first order conditions to zero and equating 59 and 60-

$$T^{\beta}\left[x_w \cdot T^{\beta} - \frac{\beta^{\beta} \alpha w}{(1+\beta)^{1+\beta}}\right] = T^{\beta}\left[x_h \cdot T^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right] \tag{63}$$

or

$$T^{\beta}(x_h - x_w) = (1-\alpha)\frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}} \tag{64}$$

Using 63 while equating 59 and 61:

$$x_h + x_w = \frac{T\alpha w}{\beta} \tag{65}$$

Using these in the budget constraint we find that:

$$t_w = \frac{\beta - 1}{\beta + 1}$$

However, since $\beta < 1$, we must be hitting a boundary condition with respect to t_w. Checking $\frac{\partial L_N}{\partial t_w}$, we see that $\frac{\partial L_N}{\partial t_w} < 0$. Hence, $t_w = 0$ and $T = 1$. Hence, setting the derivatives of the Lagrangian with respect to x_h, x_w and λ to zero we find:

$$\begin{aligned} x_h - x_w &= (1-\alpha)\frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}} \\ x_h + x_w &= \alpha w \end{aligned}$$

Solving simultaneously for x_h and x_w:

$$\begin{aligned} x_h &= \frac{\left[\alpha(1+\beta)^{1+\beta} + (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} \\ x_w &= \frac{\left[\alpha(1+\beta)^{1+\beta} - (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} \end{aligned}$$

The indirect utility functions are hence:

$$\begin{aligned} V_h &= \frac{\left[\alpha(1+\beta)^{1+\beta} + (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} \\ V_w &= \frac{\left[\alpha(1+\beta)^{1+\beta} - (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} \end{aligned}$$

From the analysis of household decision making in the non-patriarchal regime, we learn the following about the couple's labour supply ($l_i = 1 - t_i \; where \; i = w, h$) and welfare:

$$l_h = \begin{cases} 1 & if \;\; \alpha \in (0,1) \\ \frac{1-\alpha\beta}{1+\beta} & if \;\; \alpha \in \left[1, \frac{1}{\beta}\right) \\ 0 & if \;\; \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 0 & if \;\; \alpha \in (0,\beta] \\ \frac{1-\frac{\beta}{\alpha}}{\beta+1} & if \;\; \alpha \in (\beta,1) \\ 1 & if \;\; \alpha \in [1,\infty) \end{cases}$$

$$V_h = \begin{cases} \frac{\left[(1+\beta)^{1+\beta}+(1-\alpha)\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in (0,\beta] \\ \frac{\left[(1+\alpha)^{1+\beta}+(1-\alpha)\alpha^{\beta}\right]\beta^{\beta}w}{2\alpha^{\beta}(1+\beta)^{1+\beta}} & if\ \alpha \in (\beta,1) \\ \frac{\left[(1+\alpha)^{1+\beta}+1-\alpha\right]\beta^{\beta}w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1,\frac{1}{\beta}\right) \\ \frac{\left[\alpha(1+\beta)^{1+\beta}+(1-\alpha)\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta},\infty\right) \end{cases}$$

$$V_w = \begin{cases} \frac{\left[(1+\beta)^{1+\beta}-(1-\alpha)\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in (0,\beta] \\ \frac{\left[(1+\alpha)^{1+\beta}-(1-\alpha)\alpha^{\beta}\right]\beta^{\beta}w}{2\alpha^{\beta}(1+\beta)^{1+\beta}} & if\ \alpha \in (\beta,1) \\ \frac{\left[(1+\alpha)^{1+\beta}-1+\alpha\right]\beta^{\beta}w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1,\frac{1}{\beta}\right) \\ \frac{\left[\alpha(1+\beta)^{1+\beta}-(1-\alpha)\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta},\infty\right) \end{cases}$$

□

A.3 Proposition 3

When household decisions are made in the patriarchal regime and the husband restricts his wife to the household sphere, the division of labour is perfectly polarized with the husband engaging in paid work full time and not committing any time to household work for the entire domain of the wife's relative wage (α). The labour supply and welfare of the wife $\{l_w, V_w\}$ and husband $\{l_h, V_h\}$ are as follows:

$$\begin{aligned} l_w &= 0 \\ V_w &= \frac{-\beta^{\beta} + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}} \end{aligned}$$

$$\begin{aligned} l_h &= 1 \\ V_h &= \frac{\beta^{\beta} + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}} \end{aligned}$$

Proof. The bargaining problem is as follows:

$$\max_{x_w, x_h, t_h} N_P = \left[x_h \cdot (T)^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right] \left[x_w \cdot (T)^{\beta}\right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1-t_h)w \\ T &= t_h + 1 \end{aligned}$$

Setting up the Lagrangian:

$$\max_{x_w, x_h} L_P = \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot T^\beta \right] + \lambda \left\{ (1-t_h)w - x_w - x_h \right\}$$

The first derivatives of the Lagrangian are as follows:

$$\frac{\partial L_P}{\partial x_w} = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda \quad (66)$$

$$\frac{\partial L_P}{\partial x_h} = T^\beta \left[x_w \cdot T^\beta \right] - \lambda \quad (67)$$

$$\frac{\partial L_P}{\partial t_h} = \beta x_h T^{\beta-1} \left[x_w \cdot T^\beta \right] + \beta x_w T^{\beta-1} \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda w \quad (68)$$

$$\frac{\partial L_P}{\partial \lambda} = (1-t_h)w - x_w - x_h \quad (69)$$

Setting the first derivatives to 0 and equating 66 and 67 we get:

$$T^\beta (x_h - x_w) = \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \quad (70)$$

Setting equations 66, 67 and 68 to zero we get:

$$x_h + x_w = \frac{Tw}{\beta}$$

Using this in the budget constraint we get:

$$t_h = \frac{\beta - 1}{\beta + 1} \quad (71)$$

Since $\beta < 1$, it must be that we cannot set $\frac{\partial L_P}{\partial t_h}$ to zero. Checking at the boundary we find that at $t_h = 0$, $\frac{\partial L_P}{\partial t_h} < 0$. Hence we have $t_h = 0$ and $T = 1$. Setting derivatives with respect to x_h, x_w and λ to zero we get:

$$x_h - x_w = \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}$$
$$x_h + x_w = w$$

Solving the above equations simultaneously for x_h and x_w we get:

$$x_h = \frac{\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}}$$
$$x_w = \frac{-\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}}$$

The couple's indirect utility functions are given by:

$$V_h = \frac{\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}}$$
$$V_w = \frac{-\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}}$$

A.3.1 Solution for household problem when the price of purchasing household time is less than both the wife's and husband's market wages, $\overline{w} < w$ and $\overline{w} < \alpha^d w$.

The household's decision making problem is as follows:

$$\max_{x_w, x_h, t_b} N = \left[x_h \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta (\alpha^d)^{1+\beta} w}{(1+\beta)^{1+\beta}} \right]$$

s.t.

$$x_w + x_h + \overline{w} t_b = (1+\alpha^d) w$$

Since both spouses earn more than the cost of hiring household help, they will work full time and purchase all the household help they need from the market. Setting up the Lagrangian to solve for household time purchased (t_b), and the wife (V_w) and husband's welfare (V_h):

$$\max_{x_w, x_h, t_b} L = \left[x_h \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta (\alpha^d)^{1+\beta} w}{(1+\beta)^{1+\beta}} \right] + \lambda \left[(1+\alpha^d) w - x_h - x_w - \overline{w} t_b \right]$$

First derivatives:

$$\frac{\partial L_N}{\partial x_w} = t_b^\beta \left[x_h \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{72}$$

$$\frac{\partial L_N}{\partial x_h} = t_b^\beta \left[x_w \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta (\alpha^d)^{1+\beta} w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{73}$$

$$\frac{\partial L_N}{\partial t_b} = \beta x_w t_b^{\beta-1} \left[x_h \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h t_b^{\beta-1} \left[x_w \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta (\alpha^d)^{1+\beta} w}{(1+\beta)^{1+\beta}} \right] - \lambda \overline{w} \tag{74}$$

$$\frac{\partial L_N}{\partial \lambda} = (1+\alpha^d) w - x_h - x_w - \overline{w} t_b \tag{75}$$

We set all the first derivatives of the Lagrangian to zero and find the following:

$$t_b^\beta (x_h - x_w) = (1 - (\alpha^d)^{1+\beta}) \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \tag{76}$$

and,

$$x_h + x_w = \frac{t_b \overline{w}}{\beta} \tag{77}$$

Using 77 in the budget constraint:

$$t_b = \frac{\beta w (1+\alpha^d)}{\overline{w}(1+\beta)}$$

Using optimal t_b in 76 and 77, and solving simultaneously for x_h and x_w :

$$x_h = \frac{1}{2}\left[(1+\alpha^d)^{1+\beta} + (1-(\alpha^d)^{1+\beta})\right]\frac{w}{(1+\alpha^d)^{\beta}(1+\beta)}$$
$$x_w = \frac{1}{2}\left[(1+\alpha^d)^{1+\beta} - (1-(\alpha^d)^{1+\beta})\right]\frac{w}{(1+\alpha^d)^{\beta}(1+\beta)}$$

We can now calculate the indirect utility functions of the wife and husband:

$$V_h = \frac{1}{2}\left[(1+\alpha^d)^{1+\beta} + (1-(\alpha^d)^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$
$$V_w = \frac{1}{2}\left[(1+\alpha^d)^{1+\beta} - (1-(\alpha^d)^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

□

A.4 Proposition 4

When household decisions are made in the patriarchal regime, the wife faces a loss of bargaining power and welfare relative to the non-patriarchal regime, whenever she is restricted to the household sphere. Furthermore, she will often not even receive the welfare that she enjoyed when she was single, $\forall\ \alpha \in (\alpha^D, \overline{\alpha}]$. Over this range of parameters, she would rationally choose to remain single if society allows that option.

Proof. To prove the above proposition we compare the welfare of the women in patriarchal societies when they are restricted to the household sphere with their welfare when single. The wife's indirect utility under the patriarchal solution when the husband chooses the gender based division of work, denoted by subscript (Pw) is:

$$V_{Pw} = \frac{[-\beta^{\beta} + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

Her utility as single and denoted by V_{Sw} is:

$$V_{Sw} = \frac{\beta^{\beta}\alpha w}{(1+\beta)^{1+\beta}}$$

Comparing the two we find that:

$$V_{Pw} - V_{Sw} = \left[\frac{(1+\beta)^{1+\beta}}{2\beta^{\beta}} - \frac{1}{2} - \alpha\right]\frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}$$

$V_{Pw} - V_{Sw} \geq 0$ iff:

$$\alpha \leq \alpha^D = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\beta^{\beta}} - 1\right]$$

Plotting α^D as a function of β shows us that α^D is increasing in β. Further, it can also be shown numerically that $\alpha^D < \overline{\alpha}$. Refer to Appendix B for the numeric analysis. □

A.5 Proposition 5

In the patriarchal regime, under the conditions that household help can be purchased from the market, the husband's wage is higher than the cost of purchasing household help, and he chooses to restrict the wife to the household sphere, the household will purchase help from the market only if the husband's wage, $w > \frac{\overline{w}}{\beta}$.
The household help purchased from the market (t_b), the welfare of the wife (V_w) and husband (V_h) as a function of the husband's wage are as follows:

$$t_b = \begin{cases} 0 & if\ w \leq \frac{\overline{w}}{\beta} \\ \frac{\frac{\beta w}{\overline{w}}-1}{1+\beta} & if\ w > \frac{\overline{w}}{\beta} \end{cases}$$

$$V_w = \begin{cases} \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\left(\frac{\beta w}{\overline{w}}\right)^{\beta}} - 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w \leq \frac{\overline{w}}{\beta} \\ \\ \frac{1}{2}\left[\left(1+\frac{\overline{w}}{w}\right)^{1+\beta} - 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w > \frac{\overline{w}}{\beta} \end{cases}$$

$$V_h = \begin{cases} \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\left(\frac{\beta w}{\overline{w}}\right)^{\beta}} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w \leq \frac{\overline{w}}{\beta} \\ \\ \frac{1}{2}\left[\left(1+\frac{\overline{w}}{w}\right)^{1+\beta} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w > \frac{\overline{w}}{\beta} \end{cases}$$

Proof. The household decision making problem is given as follows:

$$\max_{x_w, x_h, t_b} \left[x_h \cdot (T)^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot (T)^{\beta}\right]$$

s.t.

$$\begin{aligned} x_w + x_h + \overline{w} t_b &= w \\ T &= 1 + t_b \end{aligned}$$

Setting up the Lagrangian we have:

$$\max_{x_w, x_h, t_b} L_P = \left[x_h \cdot (1+t_b)^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot (1+t_b)^{\beta}\right] + \lambda\left[w - x_w - x_h - \overline{w} t_b\right]$$

First derivatives of the Lagrangian are as follows:

$$\frac{\partial L_P}{\partial x_w} = (1+t_b)^{\beta}\left[x_h \cdot (1+t_b)^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}\right] - \lambda \quad (78)$$

$$\frac{\partial L_P}{\partial x_h} = (1+t_b)^{\beta}\left[x_w \cdot (1+t_b)^{\beta}\right] - \lambda \quad (79)$$

$$\frac{\partial L_P}{\partial t_b} = \beta x_w (1+t_b)^{\beta-1}\left[x_h \cdot (1+t_b)^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}\right] + \beta x_h (1+t_b)^{\beta-1}\left[x_w \cdot (1+t_b)^{\beta}\right] - \lambda \overline{w} \quad (80)$$

$$\frac{\partial L_P}{\partial \lambda} = w - x_h - x_w - \overline{w} t_b \quad (81)$$

We set all the first derivatives of the Lagrangian to zero and find the following:

$$(1+t_b)^\beta(x_h-x_w)=\frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \tag{82}$$

and,

$$x_h+x_w=\frac{(1+t_b)\overline{w}}{\beta} \tag{83}$$

Using 83 in the budget constraint:

$$t_b=\frac{\frac{\beta w}{\overline{w}}-1}{1+\beta}$$

We can see here that the household time bought can potentially have a boundary solution i.e.

$$t_b=\begin{cases}0 & if\ w\leq\frac{\overline{w}}{\beta}\\ \frac{\frac{\beta w}{\overline{w}}-1}{1+\beta} & if\ w>\frac{\overline{w}}{\beta}\end{cases} \tag{84}$$

If t_b hits a boundary solution, it must mean that we cannot set $\frac{\partial L}{\partial t_b}=0$ given that $\frac{\partial L}{\partial t_b}<0$ at $t_b=0$, which is easily verified. We hence solve for the unknowns in both cases.

Purchased household time has a boundary solution, $w\leq\frac{\overline{w}}{\beta}$

Here we have $t_h=0$, $t_w=1$, $t_b=0$ and $T=1$. We set the derivatives of the Lagrangian with respect to x_h, x_w and λ to zero. This gives us:

$$x_h+x_w=w \tag{85}$$

and

$$x_h-x_w=\frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \tag{86}$$

Solving the linear equations, we get:

$$V_h=x_h=\frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\left(\frac{\beta w}{\overline{w}}\right)^\beta}+1\right]\frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}$$
$$V_w=x_w=\frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\left(\frac{\beta w}{\overline{w}}\right)^\beta}-1\right]\frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}$$

Purchased household time has an interior solution, $w>\frac{\overline{w}}{\beta}$

Here we have $t_h=0$, $t_w=1$, $t_b=\frac{\frac{\beta w}{\overline{w}}-1}{1+\beta}$ and $T=\frac{\beta(\frac{w}{\overline{w}}+1)}{1+\beta}$. Using this in 82 and 83:

$$x_h+x_w=\frac{\overline{w}+w}{(1+\beta)} \tag{87}$$

and

$$x_h - x_w = \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \tag{88}$$

Solving the linear equations, we get:

$$x_h = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} + 1\right] \frac{(\delta)^{\beta} w}{(1+\delta)^{\beta}(1+\beta)}$$
$$x_w = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} - 1\right] \frac{(\delta)^{\beta} w}{(1+\delta)^{\beta}(1+\beta)}$$

The indirect utility functions are given by:

$$V_h = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} + 1\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$
$$V_w = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} - 1\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

□

B Numerical Analysis

B.1 Numerical Analysis for Welfare Comparisons between the Patriarchal and Non-Patriarchal Solutions

The husband will choose to prevent his wife from joining the labour market if exercising this option improves his welfare.

B.1.1 CASE: $\alpha \in (\beta, 1)$

$$V_{Ph} - V_{Nh} = \left[\frac{(1+\beta)^{1+\beta}}{\beta^{\beta}} + \alpha - \frac{(1+\alpha)^{1+\beta}}{\alpha^{\beta}}\right] \frac{\beta^{\beta} w}{2(1+\beta)^{1+\beta}} \tag{89}$$

$V_{Ph} \geq V_{Nh}$ iff:

$$\frac{(1+\beta)^{1+\beta}}{\beta^{\beta}} + \alpha - \frac{(1+\alpha)^{1+\beta}}{\alpha^{\beta}} \geq 0 \tag{90}$$

We plot the $V_{Ph} - V_{Nh}$ for different values of β in figures 12, 13, 14 and 15. It is seen that $V_{Ph} - V_{Nh}$ never falls below 0 and is lower bounded by β.

B.1.2 CASE: $\alpha \in \left[1, \frac{1}{\beta}\right)$

$$V_{Ph} - V_{Nh} = \left[\frac{(1+\beta)^{1+\beta}}{\beta^{\beta}} + \alpha - (1+\alpha)^{\beta}\right] \frac{\beta^{\beta} w}{2(1+\beta)^{1+\beta}} \tag{91}$$

$$V_{Ph} - V_{Nh} \geq 0$$

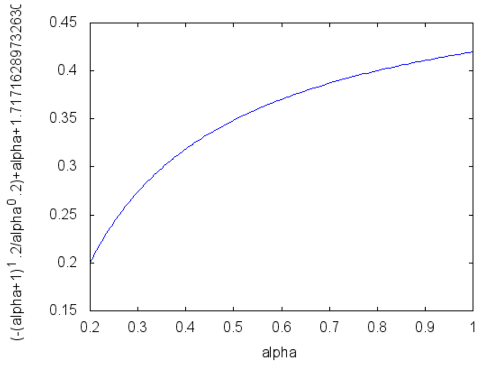

Figure 12: $V_{Ph} - V_{Nh}$ for $\beta = 0.2$ for $\alpha \in (\beta, 1)$

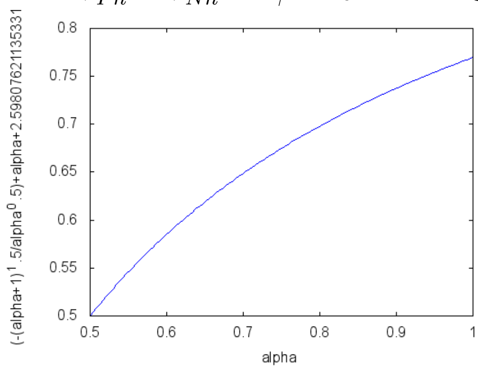

Figure 13: $V_{Ph} - V_{Nh}$ for $\beta = 0.5$ for $\alpha \in (\beta, 1)$

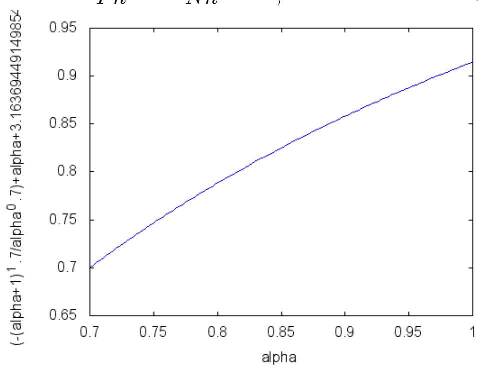

Figure 14: $V_{Ph} - V_{Nh}$ for $\beta = 0.7$ for $\alpha \in (\beta, 1)$

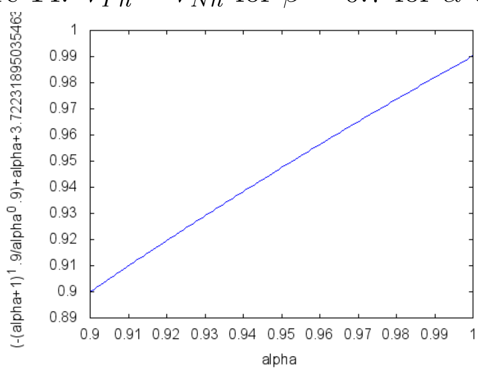

Figure 15: $V_{Ph} - V_{Nh}$ for $\beta = 0.9$ for $\alpha \in (\beta, 1)$

Iff,

$$\frac{(1+\beta)^{1+\beta}}{\beta^\beta} + \alpha - (1+\alpha)^\beta \geq 0$$

We plot $\frac{(1+\beta)^{1+\beta}}{\beta^\beta} + \alpha - (1+\alpha)^\beta$ for different values of $\beta \in (0,1)$. The numerical plots are given in figures 16, 17, 18 and 19. We observe for the range $\alpha \in \left[1, \frac{1}{\beta}\right)$, there exists an $\overline{\alpha}$ such that when $\alpha \leq \overline{\alpha}$, $V_{Ph} \geq V_{Nh}$. However,$\alpha \in \left[1, \frac{1}{\beta}\right)$ only if $\beta \in (0, 0.68)$. Further,we observe that $\overline{\alpha}$ is a decreasing function of β.

B.1.3 CASE: $\alpha \in \left[\frac{1}{\beta}, 1\right)$

$$V_{Ph} - V_{Nh} \geq 0$$

Iff,

$$\alpha \leq \overline{\alpha} = \frac{(1+\beta)^{1+\beta}}{(1+\beta)^{1+\beta} - \beta^\beta}$$

The plot of $\overline{\alpha}$ as a function of β is given in figure 20. We identify numerically that $\overline{\alpha}$ exceed $\frac{1}{\beta}$ only for $\beta \in (0.68, 1)$. Further,we observe that $\overline{\alpha}$ is a decreasing function of β.

B.2 Welfare of the Wife under the Patriarchal solution

$V_{Pw} - V_{Sw} \geq 0$ if and only if:

$$\alpha \leq \alpha^D = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\beta^\beta} - 1\right]$$

The plot of α^D as a function of β shown in figure 21.

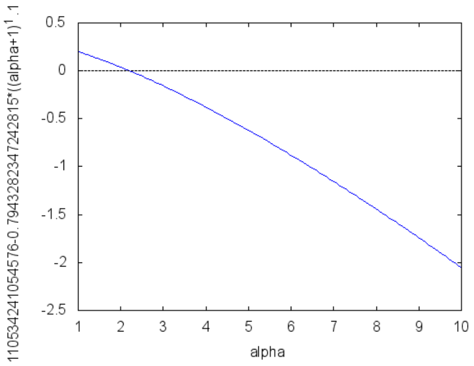

Figure 16: $V_{Ph} - V_{Nh}$ for $\beta = 0.1$, with numerically identified $\overline{\alpha} = 2.2$.

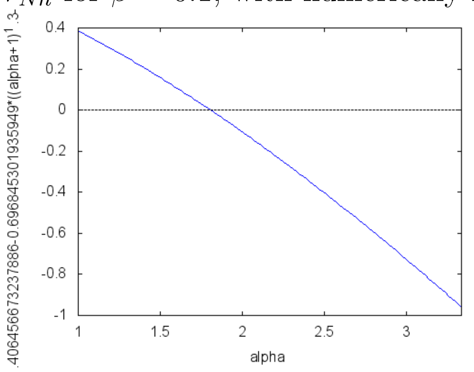

Figure 17: $V_{Ph} - V_{Nh}$ for $\beta = 0.3$ with numerically identified $\overline{\alpha} = 1.8$.

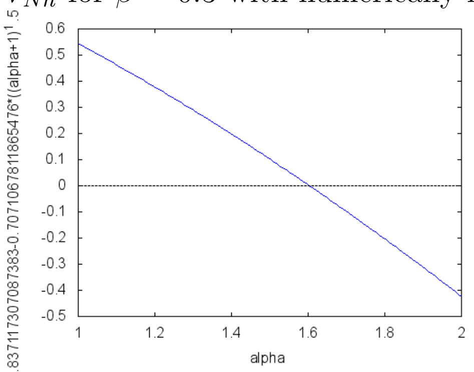

Figure 18: $V_{Ph} - V_{Nh}$ for $\beta = 0.5$ with numerically identified $\overline{\alpha} = 1.475$.

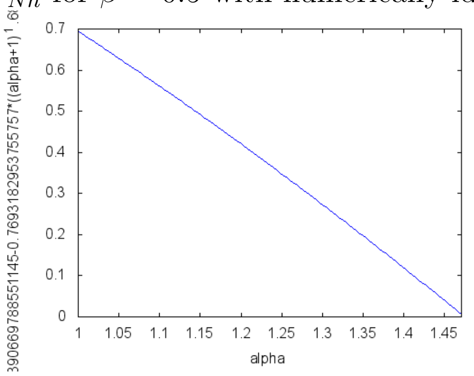

Figure 19: $V_{Ph} - V_{Nh}$ for $\beta = 0.68$ with numerically identified $\overline{\alpha}$ exceeding $\frac{1}{\beta}$ and hence dictatorship holds in the entire range.

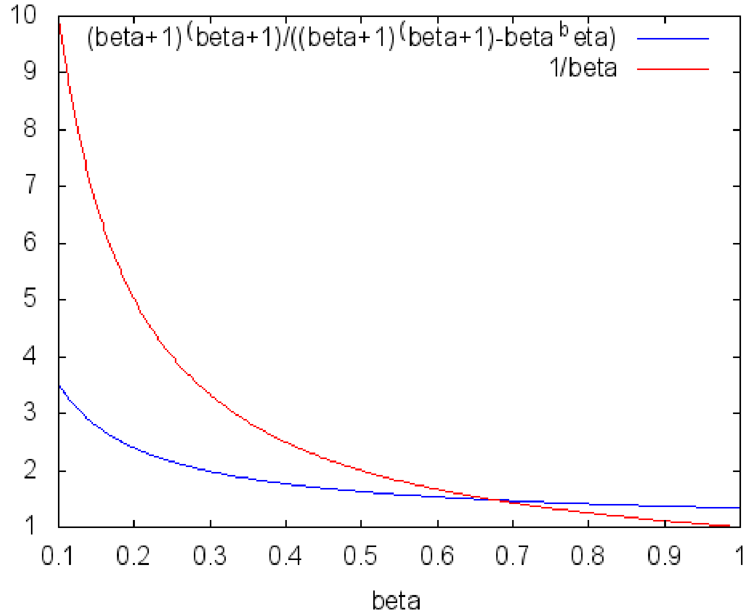

Figure 20: $\overline{\alpha}$ and β for $\beta \in (0, 1)$.

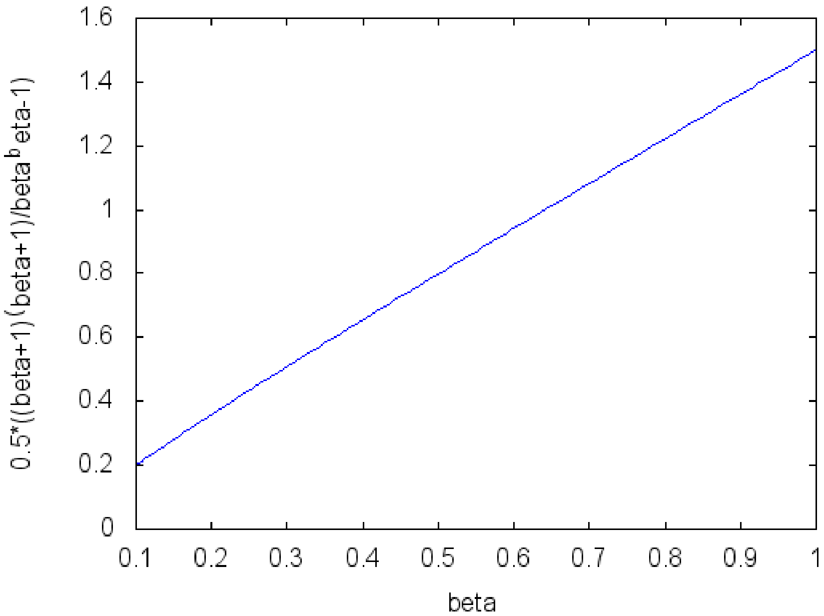

Figure 21: α^D as a function of β for $\beta \in (0, 1)$.